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**Large-time existence for one-dimensional Green-Naghdi equations with vorticity.** (English)

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**Summary:** This essay is concerned with the one-dimensional Green-Naghdi equations in the presence of a non-zero vorticity according to the derivation in [5], and with the addition of a small surface tension. The Green-Naghdi system is first rewritten as an equivalent system by using an adequate change of unknowns. We show that solutions to this model may be obtained by a standard Picard iterative scheme. No loss of regularity is involved with respect to the initial data. Moreover solutions exist at the same level of regularity as for first order hyperbolic symmetric systems, *i.e.* with a regularity in Sobolev spaces of order  $s > 3/2$ .

**MSC:**

**76B03** Existence, uniqueness, and regularity theory for incompressible inviscid fluids

**76B10** Jets and cavities, cavitation, free-streamline theory, water-entry problems, airfoil and hydrofoil theory, sloshing

**35Q31** Euler equations

**Keywords:**

shallow water Green-Naghdi equations; rotational flow; wave-current interactions; rip-currents; surface tension; existence of solutions

**Full Text:** DOI

**References:**

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