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**Single point blow-up and final profile for a perturbed nonlinear heat equation with a gradient and a non-local term.** (English) [Zbl 1479.35136](#)

Discrete Contin. Dyn. Syst., Ser. S 14, No. 8, 2607-2623 (2021).

**Summary:** We consider in this paper a perturbation of the standard semilinear heat equation by a term involving the space derivative and a non-local term. In some earlier work [1], we constructed a blow-up solution for that equation, and showed that it blows up (at least) at the origin. We also derived the so called “intermediate blow-up profile”. In this paper, we prove the single point blow-up property and determine the final blow-up profile.

**MSC:**

- [35B44](#) Blow-up in context of PDEs
- [35K15](#) Initial value problems for second-order parabolic equations
- [35K58](#) Semilinear parabolic equations
- [35R09](#) Integro-partial differential equations

**Keywords:**

nonlinear heat equation; gradient term; non-local term; final blow-up profile

**Full Text:** [DOI](#) [arXiv](#)

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