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The generalized Carrier-Greenspan transform for the shallow water system with arbitrary initial and boundary conditions. (English) Zbl 1477.35177

Water Waves 3, No. 1, 267-296 (2021).

Summary: We put forward a solution to the initial boundary value (IBV) problem for the nonlinear shallow water system in inclined channels of arbitrary cross section by means of the generalized Carrier-Greenspan hodograph transform [*A. Rybkin* et al., J. Fluid Mech. 748, 416–432 (2014; Zbl 1416.86007)]. Since the Carrier-Greenspan transform, while linearizing the shallow water system, seriously entangles the IBV in the hodograph plane, all previous solutions required some restrictive assumptions on the IBV conditions, e.g., zero initial velocity, smallness of boundary conditions. For arbitrary non-breaking initial conditions in the physical space, we present an explicit formula for equivalent IBV conditions in the hodograph plane, which can readily be treated by conventional methods. Our procedure, which we call the method of data projection, is based on the Taylor formula and allows us to reduce the transformed IBV data given on curves in the hodograph plane to the equivalent data on lines. Our method works equally well for any inclined bathymetry (not only plane beaches) and, moreover, is fully analytical for U-shaped bays. Numerical simulations show that our method is very robust and can be used to give express forecasting of tsunami wave inundation in narrow bays and fjords.

MSC:

35Q35 PDEs in connection with fluid mechanics
35L50 Initial-boundary value problems for first-order hyperbolic systems
76B03 Existence, uniqueness, and regularity theory for incompressible inviscid fluids
76B15 Water waves, gravity waves; dispersion and scattering, nonlinear interaction
35Q31 Euler equations

Keywords:

nonlinear shallow water equations; tsunami; inclined channels; Carrier-Greenspan transformation; initial boundary value problem

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