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A Hamiltonian structure of the Isobe-Kakinuma model for water waves. (English)

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Summary: We consider the Isobe-Kakinuma model for water waves, which is obtained as the system of Euler-Lagrange equations for a Lagrangian approximating Luke's Lagrangian for water waves. We show that the Isobe-Kakinuma model also enjoys a Hamiltonian structure analogous to the one exhibited by V. E. Zakharov on the full water wave problem and, moreover, that the Hamiltonian of the Isobe-Kakinuma model is a higher order shallow water approximation to the one of the full water wave problem.

MSC:

76B15 Water waves, gravity waves; dispersion and scattering, nonlinear interaction

35Q35 PDEs in connection with fluid mechanics

70H05 Hamilton's equations

Keywords:

water wave; Hamiltonian structure; shallow water approximation; Euler-Lagrange equations

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