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The essential spectrum of periodically stationary solutions of the complex Ginzburg-Landau equation. (English) Zbl 1483.35234

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Summary: We establish the existence and regularity properties of a monodromy operator for the linearization of the cubic-quintic complex Ginzburg-Landau equation about a periodically stationary (breather) solution. We derive a formula for the essential spectrum of the monodromy operator in terms of that of the associated asymptotic linear differential operator. This result is obtained using the theory of analytic semigroups under the assumption that the Ginzburg-Landau equation includes a spectral filtering (diffusion) term. We discuss applications to the stability of periodically stationary pulses in ultrafast fiber lasers.

MSC:

35Q56	Ginzburg-Landau equations
35Q55	NLS equations (nonlinear Schrödinger equations)
35Q41	Time-dependent Schrödinger equations and Dirac equations
35B10	Periodic solutions to PDEs
35B65	Smoothness and regularity of solutions to PDEs
35A01	Existence problems for PDEs: global existence, local existence, non-existence
37L15	Stability problems for infinite-dimensional dissipative dynamical systems
47D06	One-parameter semigroups and linear evolution equations
78A60	Lasers, masers, optical bistability, nonlinear optics

Keywords:

nonlinear waves; breather solutions; essential spectrum; analytic semigroups; fiber lasers

Full Text: DOI arXiv

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