Danchin, R.; **Ducomet, B.**

On the global existence for the compressible Euler-Poisson system, and the instability of static solutions. (English) Zbl 1481.35322

J. Evol. Equ. 21, No. 3, 3035-3054 (2021).

Summary: We consider the Cauchy problem for the barotropic Euler system coupled to Poisson equation, in the whole space. Our main aim is to exhibit a simple functional framework that allows to handle solutions with density going to zero at infinity, but that need not be compactly supported. We have in mind in particular the 3D static solution, when the polytropic index γ of the gas is equal to $6/5$. Our first result is the local existence of classical solutions in a simple functional framework that does not require the velocity to tend to 0 at infinity and the density to be compactly supported. Next, following the work by Grassin and Serre dedicated to the compressible Euler system [*M. Grassin* and *D. Serre*, C. R. Acad. Sci., Paris, Sér. I, Math. 325, No. 7, 721–726 (1997; Zbl 0887.35125); *M. Grassin*, Indiana Univ. Math. J. 47, No. 4, 1397–1432 (1998; Zbl 0930.35134)], we show that if the initial density is small enough, and the initial velocity is close to some reference vector field u_0 such that the spectrum of Du_0 is positive and bounded away from zero, then the corresponding classical solution is global, and satisfies algebraic time decay estimates. Compared to our recent paper [*X. Blanc*, the authors and *Š. Nečasová*, J. Hyperbolic Differ. Equ. 18, No. 1, 169–193 (2021; Zbl 1473.35396)], we are able to handle the 3D static solution that was mentioned above, and to show its instability, within our functional framework.

MSC:

35Q35	PDEs in connection with fluid mechanics
76N10	Existence, uniqueness, and regularity theory for compressible fluids and gas dynamics
35A01	Existence problems for PDEs: global existence, local existence, non-existence
35J05	Laplace operator, Helmholtz equation (reduced wave equation), Poisson equation

Keywords:

compressible Euler system; Poisson; global solution; decay; instability

Full Text: DOI

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