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Analytical and numerical temperature distribution in a 3-D triple-layer skin tissue subjected to a multi-point laser beam. (English) [Zbl 1479.78020](#)

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Summary: In the present study, the temporal and spatial variation of temperature in a three-dimensional triple-layer skin tissue under the laser heating is determined. Using the method of separation of variables along with the Laplace transform, the so-called Pennes bio-heat equation is analytically solved in a 3D triple-layer tissue in which each layer has its own thermo-physical properties. The laser heating of the skin, with both single and multiple laser beams, is modelled based on time-dependent Gaussian-shaped irradiance distributions with exponential axial attenuation. For the presented solution approach, it can be shown that the laser can be considered as an arbitrary function of time such as pulses with a specified time interval with each desired spatial distribution. Besides the analytical solution, the governing equations are solved numerically by using the standard finite element method and the results are compared with the analytical solution to investigate the effects of laser heating on human skin. The effects of using single and multiple-point laser beams on the temperature increment are investigated. A good agreement between both analytical and numerical solutions is observed. The obtained results indicate that a better temperature distribution in the skin tissue is obtained; whenever, a multi-point laser is employed.

MSC:

- 78A60 Lasers, masers, optical bistability, nonlinear optics
- 78A48 Composite media; random media in optics and electromagnetic theory
- 80A19 Diffusive and convective heat and mass transfer, heat flow
- 92C05 Biophysics
- 35Q60 PDEs in connection with optics and electromagnetic theory
- 35Q79 PDEs in connection with classical thermodynamics and heat transfer
- 35Q92 PDEs in connection with biology, chemistry and other natural sciences
- 44A10 Laplace transform
- 78M10 Finite element, Galerkin and related methods applied to problems in optics and electromagnetic theory

Keywords:

analytical solution; biological heat transfer; Laplace transform; multi-layer skin tissue; multi-point laser beam; pulsed laser; Pennes bio-heat equation

Full Text: [DOI](#)

References:

- [1] Ng, E.; Chua, L., Mesh-independent prediction of skin burns injury, *J Med Eng Technol*, 24, 255-261 (2000) · [doi:10.1080/03091900010007578](#)
- [2] Sakurai, A.; Maruyama, S.; Matsubara, K., The radiation element method coupled with the bioheat transfer equation applied to the analysis of the photothermal effect of tissues, *Numer Heat Transf Part A*, 58, 625-640 (2010) · [doi:10.1080/10407782.2010.516698](#)
- [3] Pennes, HH, Analysis of tissue and arterial blood temperatures in the resting human forearm, *J Appl Physiol*, 1, 93-122 (1948) · [doi:10.1152/jappl.1948.1.2.93](#)
- [4] Jiang, S.; Ma, N.; Li, H.; Zhang, X., Effects of thermal properties and geometrical dimensions on skin burn injuries, *Burns*, 28, 8, 713-717 (2002) · [doi:10.1016/S0305-4179\(02\)00104-3](#)
- [5] Shih, TC; Yuan, P.; Lin, WL; Kou, HS, Analytical analysis of the pennes bioheat transfer equation with sinusoidal heat flux condition on skin surface, *Med Eng Phys*, 29, 946-953 (2007) · [doi:10.1016/j.medengphy.2006.10.008](#)
- [6] Liu, J.; Xu, LX, Estimation of blood perfusion using phase shift in temperature response to sinusoidal heating at the skin surface, *IEEE Trans Biomed Eng*, 46, 1037-1043 (1999) · [doi:10.1109/10.784134](#)
- [7] Ahmadikia, H.; Fazlali, R.; Moradi, A., Analytical solution of the parabolic and hyperbolic heat transfer equations with constant and transient heat flux conditions on skin tissue, *Int Commun Heat Mass Transf*, 39, 121-130 (2012) · [doi:10.1016/j.icheatmasstransfer.2011.09.001](#)
- [8] Askarizadeh, H.; Ahmadikia, H., Analytical study on the transient heating of a two-dimensional skin tissue using parabolic and hyperbolic bioheat transfer equations, *Appl Math Model*, 39, 3704-3720 (2015) · [Zbl 1443.80012](#) · [doi:10.1016/j.apm.2014.12.003](#)

- [9] Lin, SM; Li, CY, Analytical solutions of non-fourier bio-heat conductions for skin subjected to pulsed laser heating, *Int J Therm Sci*, 110, 146-158 (2016) · doi:10.1016/j.ijthermalsci.2016.06.034
- [10] Frankel, J.; Vick, B.; Özisik, M., Flux formulation of hyperbolic heat conduction, *J Appl Phys*, 58, 3340-3345 (1985) · doi:10.1063/1.335795
- [11] Ma, J.; Sun, Y.; Yang, J., Analytical solution of dual-phase-lag heat conduction in a finite medium subjected to a moving heat source, *Int J Therm Sci*, 125, 34-43 (2018) · doi:10.1016/j.ijthermalsci.2017.11.005
- [12] Ma, J.; Yang, X.; Liu, S.; Sun, Y.; Yang, J., Exact solution of thermal response in a three-dimensional living bio-tissue subjected to a scanning laser beam, *Int J Heat Mass Transf*, 124, 1107-1116 (2018) · doi:10.1016/j.ijheatmasstransfer.2018.04.042
- [13] Alzahrani, FS; Abbas, IA, Analytical estimations of temperature in a living tissue generated by laser irradiation using experimental data, *J Therm Biol*, 85, 102421 (2019) · doi:10.1016/j.jtherbio.2019.102421
- [14] Torabi, M.; Zhang, K., Multi-dimensional dual-phase-lag heat conduction in cylindrical coordinates: analytical and numerical solutions, *Int J Heat Mass Transf*, 78, 960-966 (2014) · doi:10.1016/j.ijheatmasstransfer.2014.07.038
- [15] Kumar, S.; Srivastava, A., Finite integral transform-based analytical solutions of dual phase lag bio-heat transfer equation, *Appl Math Model*, 52, 378-403 (2017) · Zbl 1480.80021 · doi:10.1016/j.apm.2017.05.041
- [16] De Monte, F.; Haji-Sheikh, A., Bio-heat diffusion under local thermal non-equilibrium conditions using dual-phase lag-based Green's functions, *Int J Heat Mass Transf*, 113, 1291-1305 (2017) · doi:10.1016/j.ijheatmasstransfer.2017.06.006
- [17] Sun, Y.; Ma, J.; Liu, S.; Yang, J., Analytical solution of transient heat conduction in a bi-layered circular plate irradiated by laser pulse, *Can J Phys*, 95, 322-330 (2017) · doi:10.1139/cjp-2016-0603
- [18] Ramadan, K., Semi-analytical solutions for the dual phase lag heat conduction in multilayered media, *Int J Therm Sci*, 48, 14-25 (2009) · doi:10.1016/j.ijthermalsci.2008.03.004
- [19] Bagaria, HG; Johnson, DT, Transient solution to the bioheat equation and optimization for magnetic fluid hyperthermia treatment, *Int J Hyperthermia*, 21, 57-75 (2005) · doi:10.1080/02656730410001726956
- [20] Mahjoob, S.; Vafai, K., Analytical characterization of heat transport through biological media incorporating hyperthermia treatment, *Int J Heat Mass Transf*, 52, 1608-1618 (2009) · Zbl 1157.80353 · doi:10.1016/j.ijheatmasstransfer.2008.07.038
- [21] Mahjoob S, Vafai K (2010) Analysis of bioheat transport through a dual layer biological media. *J Heat Transf* 132
- [22] Weinbaum S, Jiji L, Lemons D (1984) Theory and experiment for the effect of vascular microstructure on surface tissue heat transfer—part I: Anatomical foundation and model conceptualization 106(4):321-330
- [23] Singh, R.; Das, K.; Okajima, J.; Maruyama, S.; Mishra, SC, Modeling skin cooling using optical windows and cryogenics during laser induced hyperthermia in a multilayer vascularized tissue, *Appl Therm Eng*, 89, 28-35 (2015) · doi:10.1016/j.applthermaleng.2015.06.006
- [24] Sarkar D, Haji-Sheikh A, Jain A (2015) November 13-19 theoretical analysis of transient bioheat transfer in multi-layer tissue. In: ASME international mechanical engineering congress and exposition vol 57380, p V003T03A101
- [25] Sarkar, D.; Haji-Sheikh, A.; Jain, A., Temperature distribution in multi-layer skin tissue in presence of a tumor, *Int J Heat Mass Transf*, 91, 602-610 (2015) · doi:10.1016/j.ijheatmasstransfer.2015.07.089
- [26] Shen, W.; Zhang, J.; Yang, F., Three-dimensional model on thermal response of skin subject to laser heating, *Comput Methods Biomech Biomed Eng*, 8, 115-125 (2005) · doi:10.1080/10255840500180849
- [27] Akula, SC; Maniyeri, R., Numerical simulation of bioheat transfer: a comparative study on hyperbolic and parabolic heat conduction, *J Braz Soc Mech Sci Eng*, 42, 1, 1-13 (2020) · doi:10.1007/s40430-019-2132-x
- [28] Arefmanesh, A.; Arani, AAA; Emamifar, A., Semi-analytical solutions for different non-linear models of dual phase lag equation in living tissues, *Int Commun Heat Mass Transf*, 115, 104596 (2020) · doi:10.1016/j.icheatmasstransfer.2020.104596
- [29] Becker, S.; Kuznetsov, A., Local temperature rises influence in vivo electroporation pore development: a numerical stratum corneum lipid phase transition model, *ASME J Biomech Eng*, 129, 5, 712-721 (2007) · doi:10.1115/1.2768380
- [30] Denet, AR; Vanbever, R.; Pr at, V., Skin electroporation for transdermal and topical delivery, *Adv Drug Deliv Rev*, 56, 659-674 (2004) · doi:10.1016/j.addr.2003.10.027
- [31] Pliquett, U., Mechanistic studies of molecular transdermal transport due to skin electroporation, *Adv Drug Deliv Rev*, 35, 41-60 (1999) · doi:10.1016/S0169-409X(98)00062-3
- [32] Haji-Sheikh, A.; Beck, J.; Agonafer, D., Steady-state heat conduction in multi-layer bodies, *Int J Heat Mass Transf*, 46, 2363-2379 (2003) · Zbl 1033.80002 · doi:10.1016/S0017-9310(02)00542-2
- [33] Geer J, Desai A, Sammakia B (2007) Heat conduction in multilayered rectangular domains
- [34] Choobineh, L.; Jain, A., Analytical solution for steady-state and transient temperature fields in vertically stacked 3-d integrated circuits, *IEEE Trans Compon Packaging Manuf Technol*, 2, 2031-2039 (2012) · doi:10.1109/TCPMT.2012.2213820
- [35] Choobineh, L.; Jain, A., An explicit analytical model for rapid computation of temperature field in a three-dimensional integrated circuit (3d ic), *Int J Therm Sci*, 87, 103-109 (2015) · doi:10.1016/j.ijthermalsci.2014.08.012
- [36] Sheils, NE; Deconinck, B., Initial-to-interface maps for the heat equation on composite domains, *Stud Appl Math*, 137, 140-154 (2016) · Zbl 1346.35195 · doi:10.1111/sapm.12138
- [37] Durkee, J. Jr; Antich, P., Exact solutions to the multi-region time-dependent bioheat equation with transient heat sources and boundary conditions, *Phys Med Biol*, 36, 345 (1991) · doi:10.1088/0031-9155/36/3/004
- [38] Swinehart, DF, The Beer-Lambert law, *J Chem Educ*, 39, 7, 333 (1962) · doi:10.1021/ed039p333
- [39] Hickson, R.; Barry, S.; Mercer, G., Critical times in multilayer diffusion. Part 1: exact solutions, *Int J Heat Mass Transf*, 52, 5776-5783 (2009) · Zbl 1177.80022 · doi:10.1016/j.ijheatmasstransfer.2009.08.013

- [40] Becker, S., One-dimensional transient heat conduction in composite living perfuse tissue, *ASME J Heat Transf*, 135, 7, 071002 (2013) · [doi:10.1115/1.4024063](https://doi.org/10.1115/1.4024063)
- [41] Mikhailov, MD; Özişik, MN; Vulchanov, NL, Diffusion in composite layers with automatic solution of the eigenvalue problem, *Int J Heat Mass Transf*, 26, 1131-1141 (1983) · [Zbl 0537.76059](https://zbmath.org/journals/ijhmt/26/1131.html) · [doi:10.1016/S0017-9310\(83\)80167-7](https://doi.org/10.1016/S0017-9310(83)80167-7)
- [42] Moritz, AR; Henriques, F. Jr, Studies of thermal injury: II. The relative importance of time and surface temperature in the causation of cutaneous burns, *Am J Pathol*, 23, 5, 695 (1947)
- [43] Panjehpour, M.; Overholt, BF; Milligan, AJ; Swaggerty, MW; Wilkinson, JE; Klebanow, ER, Nd: YAG laser-induced interstitial hyperthermia using a long frosted contact probe, *Lasers Surg Med*, 10, 16-24 (1990) · [doi:10.1002/lsm.1900100106](https://doi.org/10.1002/lsm.1900100106)

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