zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Liu, Zhihui; Qiao, Zhonghua

Strong approximation of monotone stochastic partial differential equations driven by multiplicative noise. (English) Zbl 1476.65243

Stoch. Partial Differ. Equ., Anal. Comput. 9, No. 3, 559-602 (2021).

Summary: We establish a general theory of optimal strong error estimation for numerical approximations of a second-order parabolic stochastic partial differential equation with monotone drift driven by a multiplicative infinite-dimensional Wiener process. The equation is spatially discretized by Galerkin methods and temporally discretized by drift-implicit Euler and Milstein schemes. By the monotone and Lyapunov assumptions, we use both the variational and semigroup approaches to derive a spatial Sobolev regularity under the $L_\omega^p L_t^\infty \dot{H}^{1+\gamma}$-norm and a temporal Hölder regularity under the $L_\omega^p L_x^2$-norm for the solution of the proposed equation with an $\dot{H}^{1+\gamma}$-valued initial datum for $\gamma \in [0,1]$. Then we make full use of the monotonicity of the equation and tools from stochastic calculus to derive the sharp strong convergence rates $\mathscr{O}(h^{1+\gamma}+\tau^{1/2})$ and $\mathscr{O}(h^{1+\gamma}+\tau^{(1+\gamma)/2})$ for the Galerkin-based Euler and Milstein schemes, respectively.

MSC:

65M60 Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs

60H15 Stochastic partial differential equations (aspects of stochastic analysis)

60H35 Computational methods for stochastic equations (aspects of stochastic analysis)

Keywords:

monotone stochastic partial differential equation; stochastic Allen-Cahn equation; Galerkin finite element method; Euler scheme; Milstein scheme

Full Text: DOI arXiv

References:

[1] Bayer, C.; Friz, P.; Riedel, S.; Schoenmakers, J., From rough path estimates to multilevel Monte Carlo, SIAM J. Numer. Anal., 54, 3, 1449-1483 (2016) · Zbl 1343.60097 · doi:10.1137/140995209

[2] Becker, S., Gess, B., Jentzen, A., Kloeden, P.: Strong convergence rates for explicit space-time discrete numerical approximations of stochastic Allen-Cahn equations arXiv:1711.02423 · Zbl 1472.60111

[3] Becker, S.; Jentzen, A., Strong convergence rates for nonlinearity-truncated Euler-type approximations of stochastic Ginzburg-Landau equations, Stoch. Process. Appl., 129, 1, 28-69 (2019) · Zbl 1403.60052 · doi:10.1016/j.spa.2018.02.008

[4] Bréhier, C-E; Cui, J.; Hong, J., Strong convergence rates of semidiscrete splitting approximations for the stochastic Allen-Cahn equation, IMA J. Numer. Anal., 39, 4, 2096-2134 (2019) · Zbl 07130817 · doi:10.1093/imanum/dry052

[5] Cao, Y.; Hong, J.; Liu, Z., Approximating stochastic evolution equations with additive white and rough noises, SIAM J. Numer. Anal., 55, 4, 1958-1981 (2017) · Zbl 1373.60124 · doi:10.1137/16M1056122

[6] Cui, J.; Hong, J., Strong and weak convergence rates of a spatial approximation for stochastic partial differential equation with one-sided Lipschitz coefficient, SIAM J. Numer. Anal., 57, 4, 1815-1841 (2019) · Zbl 1420.60093 · doi:10.1137/18M1215554

[7] Cui, J.; Hong, J.; Liu, Z., Strong convergence rate of finite difference approximations for stochastic cubic Schrödinger equations, J. Differ. Equ., 263, 3687-3713 (2017) · Zbl 1371.60122 · doi:10.1016/j.jde.2017.05.002

[8] Cui, J.; Hong, J.; Liu, Z.; Zhou, W., Strong convergence rate of splitting schemes for stochastic nonlinear Schrödinger equations, J. Differ. Equ., 266, 9, 5625-5663 (2019) · Zbl 1431.60066 · doi:10.1016/j.jde.2018.10.034

[9] Dörsek, P., Semigroup splitting and cubature approximations for the stochastic Navier-Stokes equations, SIAM J. Numer. Anal., 50, 2, 729-746 (2012) · Zbl 1247.60090 · doi:10.1137/110833841

[10] Feng, X.; Li, Y.; Zhang, Y., Finite element methods for the stochastic Allen-Cahn equation with gradient-type multiplicative noises, SIAM J. Numer. Anal., 55, 1, 194-216 (2017) · Zbl 1357.65010 · doi:10.1137/15M1022124

[11] Gyöngy, I.; Millet, A., Rate of convergence of space time approximations for stochastic evolution equations, Potential Anal., 30, 1, 29-64 (2009) · Zbl 1168.60025 · doi:10.1007/s11118-008-9105-5

[12] Higham, D.; Mao, X.; Stuart, A., Strong convergence of Euler-type methods for nonlinear stochastic differential equation, SIAM J. Numer. Anal., 40, 3, 1041-1063 (2002) · Zbl 1026.65003 · doi:10.1137/S0036142901389530

[13] Hong, J.; Huang, C.; Liu, Z., Optimal regularity of stochastic evolution equations in M-type 2 Banach spaces, J. Differ. Equ., 267, 3, 1955-1971 (2019) · Zbl 1412.60090 · doi:10.1016/j.jde.2019.03.003

[14] Hong, J.; Liu, Z., Well-posedness and optimal regularity of stochastic evolution equations with multiplicative noises, J. Differ. Equ., 266, 8, 4712-4745 (2019) · Zbl 1418.60098 · doi:10.1016/j.jde.2018.10.005

[15] Hutzenthaler, M., Jentzen, A.: On a perturbation theory and on strong convergence rates for stochastic ordinary and partial differential equations with non-globally monotone coefficients. Ann. Probab. (arXiv:1401.0295) · Zbl 07206753

[16] Hutzenthaler, M.; Jentzen, A., Convergence of the stochastic Euler scheme for locally Lipschitz coefficients, Found. Comput. Math., 11, 6, 657-706 (2011) · Zbl 1253.65006 · doi:10.1007/s10208-011-9101-9

[17] Jentzen, A.; Röckner, M., A Milstein scheme for SPDEs, Found. Comput. Math., 15, 2, 313-362 (2015) · Zbl 1318.60072 · doi:10.1007/s10208-015-9247-y

[18] Kovács, M.; Larsson, S.; Lindgren, F., On the backward Euler approximation of the stochastic Allen-Cahn equation, J. Appl. Probab., 52, 2, 323-338 (2015) · Zbl 1323.60089 · doi:10.1239/jap/1437658601

[19] Kovács, M.; Larsson, S.; Lindgren, F., On the discretisation in time of the stochastic Allen-Cahn equation, Math. Nachr., 291, 5-6, 966-995 (2018) · Zbl 1388.60113 · doi:10.1002/mana.201600283

[20] Kruse, R., Consistency and stability of a Milstein-Galerkin finite element scheme for semilinear SPDE, Stoch. Partial Differ. Equ. Anal. Comput., 2, 4, 471-516 (2014) · Zbl 1316.60106

[21] Kruse, R., Optimal error estimates of Galerkin finite element methods for stochastic partial differential equations with multiplicative noise, IMA J. Numer. Anal., 34, 1, 217-251 (2014) · Zbl 1282.65021 · doi:10.1093/imanum/drs055

[22] Krylov, N., A \(W^n_2\)-theory of the Dirichlet problem for SPDEs in general smooth domains, Probab. Theory Relat. Fields, 98, 3, 389-421 (1994) · Zbl 0794.60062 · doi:10.1007/BF01192260

[23] Liu, J., Order of convergence of splitting schemes for both deterministic and stochastic nonlinear Schrödinger equations, SIAM J. Numer. Anal., 51, 4, 1911-1932 (2013) · Zbl 1274.60220 · doi:10.1137/12088416X

[24] Liu, W., Well-posedness of stochastic partial differential equations with Lyapunov condition, J. Differ. Equ., 255, 3, 572-592 (2013) · Zbl 1284.60130 · doi:10.1016/j.jde.2013.04.021

[25] Liu, W.; Röckner, M., Stochastic Partial Differential Equations: An Introduction (2015), Cham: Universitext, Springer, Cham · Zbl 1361.60002 · doi:10.1007/978-3-319-22354-4

[26] Liu, Z.; Qiao, Z., Wong-Zakai approximation of stochastic Allen-Cahn equation, Int. J. Numer. Anal. Model., 16, 5, 681-694 (2019) · Zbl 1431.60067

[27] Liu, Z.; Qiao, Z., Strong approximation of stochastic Allen-Cahn equation with white noise, IMA J. Numer. Anal., 40, 2, 1074-1093 (2020) · Zbl 1464.65148 · doi:10.1093/imanum/dry088

[28] Majee, A.; Prohl, A., Optimal strong rates of convergence for a space-time discretization of the stochastic Allen-Cahn equation with multiplicative noise, Comput. Methods Appl. Math., 18, 2, 297-311 (2018) · Zbl 1404.65178 · doi:10.1515/cmam-2017-0023

[29] Marinelli, C.; Röckner, M., On the maximal inequalities of Burkholder, Davis and Gundy, Expo. Math., 34, 1, 1-26 (2016) · Zbl 1335.60064 · doi:10.1016/j.exmath.2015.01.002

[30] Müller-Gronbach, T.; Ritter, K., Lower bounds and nonuniform time discretization for approximation of stochastic heat equations, Found. Comput. Math., 7, 2, 135-181 (2007) · Zbl 1136.60044 · doi:10.1007/s10208-005-0166-6

[31] Neelima, Šiška, D.: \(l^p\)-estimates and regularity for SPDEs with monotone semilinearity. Stoch. Partial Differ. Equ. Anal. Comput. 8(2), 422-459 (2020) · Zbl 1446.60048

[32] Pazy, A., Semigroups of Linear Operators and Applications to Partial Differential Equations, Applied Mathematical Sciences (1983), New York: Springer, New York · Zbl 0516.47023

[33] Prohl, A.: Strong rates of convergence for a space-time discretization of the stochastic Allen-Cahn equation with multiplicative noise (2014, preprint). https://na.uni-tuebingen.de/preprints

[34] Qi, R.; Wang, X., Optimal error estimates of Galerkin finite element methods for stochastic Allen-Cahn equation with additive noise, J. Sci. Comput., 80, 2, 1171-1194 (2019) · Zbl 1418.60103 · doi:10.1007/s10915-019-00973-8

[35] Wang, X.: An efficient explicit full discrete scheme for strong approximation of stochastic Allen-Cahn equation. arXiv:1802.09413 · Zbl 07243121