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Direct versus iterative methods for forward-backward diffusion equations. Numerical comparisons. (English) Zbl 1476.65181

SeMA J. 78, No. 3, 271-286 (2021).

Summary: By far, the standard implementation of finite difference schemes for forward-backward partial differential equations consists in employing an iterative method. This paper collects a series of numerical results which demonstrate that a direct implementation can reduce the computing time. An effective way of choosing the seed for the iterative method naturally arises.

MSC:

65M06	Finite difference methods for initial value and initial-boundary value problems involving PDEs	Cited in **1** Document
35K65	Degenerate parabolic equations	
35Q84	Fokker-Planck equations	
78M20	Finite difference methods applied to problems in optics and electromagnetic theory	
78A35	Motion of charged particles	

Keywords:

forward-backward partial differential equation; two-way partial differential equation; finite difference method; Fokker-Planck equation; direct method; iterative method

Full Text: DOI

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