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The unique continuation property for second order evolution PDEs. (English) Zbl 1480.35080
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Summary: We present a simple and self-contained approach to establish the unique continuation property for some classical evolution equations of second order in a cylindrical domain. We namely discuss this property for wave, parabolic and Schrödinger operators with time-independent principal part. Our method is builds on two-parameter Carleman inequalities combined with unique continuation across a pseudoconvex hypersurface with respect to the space variable. The most results we demonstrate in this work are more or less classical. Some of them are not stated exactly as in their original form.

MSC:

35B60 Continuation and prolongation of solutions to PDEs

35A23 Inequalities applied to PDEs involving derivatives, differential and integral operators, or integrals

35J15 Second-order elliptic equations

35K10 Second-order parabolic equations

35L10 Second-order hyperbolic equations

Keywords:

wave equation; parabolic equation; Schrödinger equation; elliptic equation; Carleman inequality; pseudoconvexity condition; non characteristic hypersurface; property of unique continuation; observability inequality

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