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To solving the heat equation with fractional load. (English) Zbl 1480.35393

Lobachevskii J. Math. 42, No. 12, 2854-2866 (2021).

Summary: In the paper, a boundary value problem for a fractionally loaded heat equations is considered in the first quadrant. The questions of the existence and uniqueness of the solution are investigated in the class of continuous functions. The loaded term has the form of the Caputo fractional derivative with respect to the spatial variable, and, the order of the derivative in the loaded term is less than the order of the differential part. The study is based on reducing the boundary value problem to a Volterra integral equation of the second kind. The kernel of the obtained integral equation contains a special function, namely, the generalized hypergeometric series. It is shown that the existence and uniqueness of solutions to the integral equation depends both on the order of the fractional derivative in the loaded term of the initial boundary value problem and on the behavior character of the load.

MSC:

35R11 Fractional partial differential equations

35K20 Initial-boundary value problems for second-order parabolic equations

Keywords:

loaded equation; Caputo fractional derivative; heat equation; Volterra integral equation

Full Text: [DOI](#)

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