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**A multi-scale Gaussian beam parametrix for the wave equation: the Dirichlet boundary value problem.** (English) Zbl 1480.35027

*J. Differ. Equations* 309, 949-993 (2022).

**Summary:** We present a construction of a multi-scale Gaussian beam parametrix for the Dirichlet boundary value problem associated with the wave equation, and study its convergence rate to the true solution in the highly oscillatory regime. The construction elaborates on the wave-atom parametrix of Bao, Qian, Ying, and Zhang and extends to a multi-scale setting the technique of Gaussian beam propagation from a boundary of Katchalov, Kurylev and Lassas.

**MSC:**

[35B40](#) Asymptotic behavior of solutions to PDEs

[35C10](#) Series solutions to PDEs

[35L05](#) Wave equation

[35L20](#) Initial-boundary value problems for second-order hyperbolic equations

[35S05](#) Pseudodifferential operators as generalizations of partial differential operators

[42C15](#) General harmonic expansions, frames

Cited in 1 Document

**Keywords:**

Gaussian beam; wave-atom; parametrix; highly oscillatory regime

**Full Text:** [DOI](#) [arXiv](#)

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