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**On group symmetries of the hydrodynamic equations for rarefied gas.** (English)

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Summary: The invariant group transformations of three-dimensional hydrodynamic equations derived from the Boltzmann equation are studied. Three levels (with respect to the Knudsen number) of hydrodynamic description are considered and compared: (a) Euler equations, (b) Navier-Stokes equations, (c) Generalized Burnett equations (GBEs), which replace the original (ill-posed) Burnett equations. The main attention is paid to group analysis of GBEs in their most general formulation because this and related questions have not been studied before in the literature. The results of group analysis of GBEs and, for comparison, of similar results for Euler and Navier-Stokes equations are presented in two theorems and discussed in detail. It is remarkable that the use of computer code greatly simplifies the proof of the results for GBEs, which are very cumbersome equations with many undetermined parameters.

**MSC:**

**35Q35** PDEs in connection with fluid mechanics

**76P05** Rarefied gas flows, Boltzmann equation in fluid mechanics

**76M60** Symmetry analysis, Lie group and Lie algebra methods applied to problems in fluid mechanics

**Keywords:**

generalized Burnett equations; Euler equations; Navier-Stokes equations; admitted Lie group; group classification

**Full Text:** DOI

**References:**

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