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An analysis to a model of tornado. (English) Zbl 07447198

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Summary: Tornado is a destructive catastrophe. We use compressible isentropic Euler equations to describe this problem. A cylindrically symmetric special solution moving with a constant velocity in $\mathbb{R}^3$ is given. It depicts how the vorticity function of the flow evolves. Even if the initial inward velocity and acceleration are both very small, the inward velocity could become very large and the vorticity could increase drastically in later time, and most of mass concentrates on a neighborhood of the moving center axis at this time. For this solution, cases when $\gamma \neq 2$ and when $\gamma = 2$ (shallow water) have some differences, while their evolution dynamics are basically the same. When $\gamma = 2$, the initial vorticity could depend on the space variables.

MSC:

35Qxx Partial differential equations of mathematical physics and other areas of application
35L60 First-order nonlinear hyperbolic equations
49L25 Viscosity solutions to Hamilton-Jacobi equations in optimal control and differential games
35L67 Shocks and singularities for hyperbolic equations
35D10 Regularity of generalized solutions of PDE (MSC2000)

Keywords:

Euler equation; compressible; cylindrical symmetry; tornado; hubble velocity

Full Text: DOI

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