Berchenko-Kogan, Yakov

The entropy of the angenent torus is approximately 1.85122. (English) Zbl 07446665

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Summary: To study the singularities that appear in mean curvature flow, one must understand *self-shrinkers*, surfaces that shrink by dilations under mean curvature flow. The simplest examples of self-shrinkers are spheres and cylinders. In [Prog. Nonlinear Differ. Equ. Appl. 7, 21–38 (1992; Zbl 0762.53028)], *S. B. Angenent* constructed the first nontrivial example of a self-shrinker, a torus. A key quantity in the study of the formation of singularities is the *entropy*, defined by Colding and Minicozzi based on work of Huisken. The values of the entropy of spheres and cylinders have explicit formulas, but there is no known formula for the entropy of the Angenent torus. In this work, we numerically estimate the entropy of the Angenent torus using the discrete Euler-Lagrange equations.

MSC:

53E10 Flows related to mean curvature
70H25 Hamilton's principle
65-04 Software, source code, etc. for problems pertaining to numerical analysis
65P99 Numerical problems in dynamical systems

Keywords:

mean curvature flow; angenent torus; discrete Euler-Lagrange equations; variational integrator

Full Text: DOI arXiv

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