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Debyaoui, Mohamed Ali; Ersoy, Mehmet

Generalised Serre-Green-Naghdi equations for open channel and for natural river hydraulics. (English) Zbl 07446496

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Summary: In this paper, we present a new non-linear dispersive model for open channel and river flows. These equations are the second-order shallow water approximation of the section-averaged (three-dimensional) incompressible and irrotational Euler system. This new asymptotic model generalises the well-known one-dimensional Serre-Green-Naghdi (SGN) equations for rectangular section on uneven bottom to arbitrary channel/river section.

MSC:

35Qxx Partial differential equations of mathematical physics and other areas of application

Keywords:

open channel flow; river flow; Euler equations; asymptotic approximation; Serre-Green-Naghdi equations; free surface shallow water equations; non-hydrostatic pressure; dispersive

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