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On an inverse problem of nonlinear imaging with fractional damping. (English)

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Summary: This paper considers the attenuated Westervelt equation in pressure formulation. The attenuation is by various models proposed in the literature and characterised by the inclusion of non-local operators that give power law damping as opposed to the exponential of classical models. The goal is the inverse problem of recovering a spatially dependent coefficient in the equation, the parameter of nonlinearity $\kappa(x)$, in what becomes a nonlinear hyperbolic equation with non-local terms. The overposed measured data is a time trace taken on a subset of the domain or its boundary. We shall show injectivity of the linearised map from κ to the overposed data and from this basis develop and analyse Newton-type schemes for its effective recovery.

MSC:

35R30 Inverse problems for PDEs
35R11 Fractional partial differential equations
35L20 Initial-boundary value problems for second-order hyperbolic equations
35L72 Second-order quasilinear hyperbolic equations
78A46 Inverse problems (including inverse scattering) in optics and electromagnetic theory

Keywords:

damped nonlinear wave equation; ultrasound; attenuated Westervelt equation in pressure formulation

Full Text: DOI arXiv

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