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Hamiltonian regularisation of the unidimensional barotropic Euler equations. (English)

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Summary: Recently, a Hamiltonian regularised shallow water (Saint-Venant) system has been introduced by Clamond and Dutykh (2018). This system is Galilean invariant, linearly non-dispersive and conserves formally an H^1-like energy. In this paper, we extend this regularisation in two directions. First, we consider the more general barotropic Euler system, the shallow water equations being formally a very special case. Second, we introduce a class regularisations, showing thus that this Hamiltonian regularisation of Clamond and Dutykh (2018) is not unique. Considering the high-frequency approximation of this regularisation, we obtain a new two-component Hunter-Saxton system. We prove that both systems – the regularised barotropic Euler system and the two-component Hunter-Saxton system – are locally (in time) well-posed, and, if singularities appear in finite time, they are necessary in the first derivatives.

MSC:

35Qxx Partial differential equations of mathematical physics and other areas of application
35Lxx Hyperbolic equations and hyperbolic systems
35Bxx Qualitative properties of solutions to partial differential equations

Keywords:

barotropic flows; Euler system; nonlinear hyperbolic systems; generalised Hunter-Saxton system; regularisation; energy conservation

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