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Longtime dynamics for a nonlinear viscoelastic equation with time-dependent memory kernel. (English) [Zbl 1479.35118](#)

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Summary: This paper investigates the well-posedness, the existence and the regularity of the time-dependent global attractor for a viscoelastic equation in $\Omega \subset \mathbb{R}^3$:

$$|\partial_t u|^\rho \partial_{tt} u - \partial_{tt} \Delta u - h_t(0) \Delta u - \int_0^\infty \partial_s h_t(s) \Delta u(t-s) \, ds + f(u) = h$$

with time-dependent memory kernel which is used to model aging phenomena of the material. By using the novel theory framework recently developed in literature [*M. Conti et al., J. Differ. Equations* 264, No. 7, 4235–4259 (2018; [Zbl 1391.35282](#)); *Am. J. Math.* 140, No. 2, 349–389 (2018; [Zbl 1480.45010](#))] and establishing some delicate integration estimates along the trajectory of the solutions in the time-dependent phase space, we show that when $\rho \in (1, 4]$, the growth exponent p of $f(u)$ is up to the critical range $1 \leq p \leq 5$, and the time-dependent memory kernel satisfies the same conditions as in [loc. cit.], the model is well-posed. Especially, when $\rho \in (1, 4)$ and $1 \leq p < 5$, the related process has an invariant time-dependent global attractor which has optimal regularity.

MSC:

- [35B40](#) Asymptotic behavior of solutions to PDEs
- [35B41](#) Attractors
- [35L20](#) Initial-boundary value problems for second-order hyperbolic equations
- [35L72](#) Second-order quasilinear hyperbolic equations
- [35R09](#) Integro-partial differential equations
- [74D10](#) Nonlinear constitutive equations for materials with memory

Keywords:

viscoelastic equation; time-dependent memory kernel; well-posedness; time-dependent global attractors; regularity of attractors

Full Text: [DOI](#)

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