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**Heat flow on time-dependent manifolds.** (English) Zbl 1479.35509

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**Summary:** We establish effective existence and uniqueness for the heat flow on time-dependent Riemannian manifolds, under minimal assumptions tailored towards the study of Ricci flow through singularities. The main point is that our estimates only depend on an upper bound for the logarithmic derivative of the volume measure. In particular, our estimates hold for any Ricci flow with scalar curvature bounded below, and such a lower bound of course depends only on the initial data.

**MSC:**

**35K15** Initial value problems for second-order parabolic equations

**35R01** PDEs on manifolds

**53E20** Ricci flows

**58J35** Heat and other parabolic equation methods for PDEs on manifolds

**Keywords:**

heat flow; Ricci flow; implicit Euler scheme.

**Full Text:** [DOI](#) [arXiv](#)

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