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Fractional Kirchhoff-Choquard equation involving Schrödinger term and upper critical exponent. (English) [Zbl 1480.35229](#)

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Summary: In this paper, we consider fractional degenerate and non-degenerate Kirchhoff type Schrödinger-Choquard problems with upper critical exponent, respectively. By studying the solutions of limit problems for above problems and establishing some local and global compactness results, we provide some sufficient conditions under which above problems have at least one or two bounded state solutions. Our main tools adopted in our proof are splitting theorem and linking theorem.

MSC:

[35J62](#) Quasilinear elliptic equations

[35R11](#) Fractional partial differential equations

[35A01](#) Existence problems for PDEs: global existence, local existence, non-existence

[35J20](#) Variational methods for second-order elliptic equations

Keywords:

fractional Kirchhoff-type Schrödinger-Choquard equations; upper critical exponent; existence

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