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A PML finite element method for electromagnetic scattering problems in a two-layer medium. (English) Zbl 1479.35842

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Summary: The paper concerns the numerical solution for three-dimensional electromagnetic scattering problems in a two-layer medium. The Cartesian perfectly matched layer (PML) method is adopted to truncate the unbounded physical domain into a bounded computational domain. Although the PML method has been used widely to solve electromagnetic scattering problems, rigorous finite element error analyses are still rare in the literature, particularly, for electromagnetic scattering problems in layered media. This paper presents a thorough error analysis for finite element approximation to the scattering problems in a two-layer medium with PML boundary condition. Numerical experiments are presented to demonstrate the efficiency of the PML method and the optimal convergence of the finite element solution.

MSC:

35Q60 PDEs in connection with optics and electromagnetic theory
78A45 Diffraction, scattering
78A48 Composite media; random media in optics and electromagnetic theory
78M10 Finite element, Galerkin and related methods applied to problems in optics and electromagnetic theory
65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
65N15 Error bounds for boundary value problems involving PDEs
65N12 Stability and convergence of numerical methods for boundary value problems involving PDEs

Keywords:

electromagnetic scattering problem; two-layer medium; Cartesian perfectly matched layer; finite element method; optimal error estimates

Full Text: DOI

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