zbMATH Open

THE FIRST RESOURCE FOR MATHEMATICS

Wei, Ting; Xian, Jun

Determining a time-dependent coefficient in a time-fractional diffusion-wave equation with the Caputo derivative by an additional integral condition. (English) Zbl 1479.35957

J. Comput. Appl. Math. 404, Article ID 113910, 22 p. (2022).

Summary: This paper is devoted to recovering a time-dependent zeroth-order coefficient in a time-fractional diffusion-wave equation with the time Caputo derivative from an additional integral condition. The uniqueness and a conditional stability for such an inverse problem are proved. Then the two-point gradient method is used to solve the inverse zeroth-order coefficient problem numerically. Some properties of the forward operator are obtained, such as the Fréchet differentiability, the Lipschitz continuity and the tangential cone condition to guarantee the convergence of the proposed algorithm. Four numerical examples in one-dimensional and two-dimensional spaces are provided to show the effectiveness and stability of the suggested algorithm.

MSC:

35R30 Inverse problems for PDEs

35L20 Initial-boundary value problems for second-order hyperbolic equations

35R11 Fractional partial differential equations

65M32 Numerical methods for inverse problems for initial value and initial-boundary value problems involving PDEs

Keywords:

time-fractional diffusion-wave equation; identification of zeroth-order coefficient; uniqueness and conditional stability; two-point gradient method; convergence analysis

Full Text: DOI

References:

[1] Sokolov, I. M.; Klafter, J., From diffusion to anomalous diffusion: A century after Einsteins Brownian motion, Chaos, 15, 2, 26103 (2005) · Zbl 1080.82022

[2] Metzler, R.; Klafter, J., Subdiffusive transport close to thermal equilibrium: from the Langevin equation to fractional diffusion, Phys. Rev. E, 61, 6 Pt A, 6308-6311 (2000)

[3] Henry, B. I.; Langlands, T. A.; Wearne, S. L., Fractional cable models for spiny neuronal dendrites, Phys. Rev. Lett., 100, 12, Article 128103 pp. (2008)

[4] Metzler, R.; Klafter, J., Boundary value problems for fractional diffusion equations, Phys. A, 278, 1-2, 107-125 (2000)

[5] Metzler, R.; Klafter, J., The random walk's guide to anomalous diffusion: a fractional dynamics approach, Phys. Rep., 339, 1, 1-77 (2000) · Zbl 0984.82032

[6] Raberto, M.; Scalas, E.; Mainardi, F., Waiting-times and returns in high-frequency financial data: an empirical study, Phys. A, 314, 1-4, 749-755 (2002) · Zbl 1001.91033

[7] Yuste, S. B.; Acedo, L.; Lindenberg, K., Reaction front in an \(a + b \to c\) reaction-subdiffusion process, Phys. Rev. E (3), 69, 3 Pt 2, Article 036126 pp. (2004)

[8] Yuste, S. B.; Lindenberg, K.; Ruiz-Lorenzo, J. J., Subdiffusion-limited reactions, Chem. Phys., 284, 1-2, 169-180 (2002)

[9] Agrawal, O. P., Solution for a fractional diffusion-wave equation defined in a bounded domain, Nonlinear Dynam., 29, 1-4, 145-155 (2002), Fractional order calculus and its applications · Zbl 1009.65085

[10] Chen, A.; Li, C. P., Numerical solution of fractional diffusion-wave equation, Numer. Funct. Anal. Optim., 37, 1, 19-39 (2016) · Zbl 1382.65236

[11] Du, R.; Cao, W. R.; Sun, Z. Z., A compact difference scheme for the fractional diffusion-wave equation, Appl. Math. Model., 34, 10, 2998-3007 (2010) · Zbl 1201.65154

[12] Sakamoto, K.; Yamamoto, M., Initial value/boundary value problems for fractional diffusion-wave equations and applications to some inverse problems, J. Math. Anal. Appl., 382, 1, 426-447 (2011) · Zbl 1219.35367

[13] Ren, J. C.; Sun, Z. Z., Numerical algorithm with high spatial accuracy for the fractional diffusion-wave equation with Neumann boundary conditions, J. Sci. Comput., 56, 2, 381-408 (2013) · Zbl 1281.65113

[14] Liu, F. W.; Meerschaert, M. M.; Mcgough, R. J.; Zhuang, P. H.; Liu, Q. X., Numerical methods for solving the multi-term

time-fractional wave-diffusion equation, Fract. Calc. Appl. Anal., 16, 1, 9-25 (2013) · Zbl 1312.65138

[15] Kian, Y.; Yamamoto, M., On existence and uniqueness of solutions for semilinear fractional wave equations, Fract. Calc. Appl. Anal., 20, 1, 117-138 (2015) · Zbl 1362.35323

[16] Dai, H. Y.; Wei, L. L.; Zhang, X. D., Numerical algorithm based on an implicit fully discrete local discontinuous Galerkin method for the fractional diffusion-wave equation, Numer. Algorithms, 67, 4, 845-862 (2014) · Zbl 1307.65130

[17] Sun, Z. Z.; Wu, X. N., A Fully Discrete Difference Scheme for a Diffusion-Wave System, 193-209 (2006), Elsevier Science Publishers B. V. · Zbl 1094.65083

[18] Hosseini, K.; Ilie, M.; Mirzazadeh, M.; Baleanu, D., An analytic study on the approximate solution of a nonlinear time-fractional Cauchy reaction-diffusion equation with the Mittag-Leffler law, Math. Methods Appl. Sci., 44, 8, 6247-6258 (2021) · Zbl 1471.35301

[19] Hosseini, K.; Ilie, M.; Mirzazadeh, M.; Yusuf, A.; Sulaiman, T. A.; Baleanu, D.; Salahshour, S., An effective computational method to deal with a time-fractional nonlinear water wave equation in the Caputo sense, Math. Comput. Simulation, 187, 248-260 (2021) · Zbl 07428957

[20] Šišková, K.; Slodička, M., Recognition of a time-dependent source in a time-fractional wave equation, Appl. Numer. Math., 121, 1-17 (2017) · Zbl 1372.65264

[21] Wei, T.; Zhang, Y., The backward problem for a time-fractional diffusion-wave equation in a bounded domain, Comput. Math. Appl., 75, 10, 3632-3648 (2018) · Zbl 1417.35224

[22] Wei, T.; Yan, X. B., Recovering a space-dependent source term in a time-fractional diffusion wave equation, J. Appl. Anal. Comput., 9, 5, 1801-1821 (2019) · Zbl 1468.65129

[23] Xian, J.; Wei, T., Determination of the initial data in a time-fractional diffusion-wave problem by a final time data, Comput. Math. Appl. (2019) · Zbl 1443.35179

[24] Liao, K. F.; Wei, T., Identifying a fractional order and a space source term in a time-fractional diffusion-wave equation simultaneously, Inverse Problems, 35, 11, Article 115002 pp. (2019) · Zbl 1442.65230

[25] Yan, X. B.; Wei, T., Determine a space-dependent source term in a time fractional diffusion-wave equation, Acta Appl. Math., 165, 1, 163-181 (2020) · Zbl 1432.35232

[26] Yan, X. B.; Zhang, Y. X.; Wei, T., Identify the fractional order and diffusion coefficient in a fractional diffusion wave equation, J. Comput. Appl. Math., 393, Article 113497 pp. (2021) · Zbl 1468.65130

[27] Wei, T.; Liao, K. F., Identifying a time-dependent zeroth-order coefficient in a time-fractional diffusion-wave equation by using the measured data at a boundary point, Appl. Anal. (2020), (in press),https://Doi.Org/10.1080/00036811.2021.1932834

[28] Berkowitz, B.; Scher, H.; Silliman, S. E., Anomalous transport in laboratory scale, heterogeneous porous media, Water Resour. Res., 36, 1, 149-158 (2000)

[29] Zheng, G. H.; Ding, M. H., Identification of the degradation coefficient for an anomalous diffusion process in hydrology, Inverse Probl., 36, 3 (2020) · Zbl 1447.65064

[30] Lopushanska, H.; Rapita, V., Inverse coefficient problem for semi-linear fractional telegraph equation, Electron. J. Differential Equations, 2015, 153, 1-13 (2015) · Zbl 1322.35177

[31] El-Borai, M. M., On the solvability of an inverse fractional abstract Cauchy problem, Int. J. Res. Rev. Appl. Sci., 4, 411-415 (2010)

[32] Lopushansky, A.; Lopushansky, O.; Sharyn, S., Nonlinear inverse problem of control diffusivity parameter determination for a space-time fractional diffusion equation, Appl. Math. Comput., 390, Article 125589 pp. (2021) · Zbl 1474.49082

[33] Fujishiro, K.; Kian, Y., Determination of time dependent factors of coefficients in fractional diffusion equations, Math. Control Relat. Fields, 6, 2, 251-269 (2016) · Zbl 1348.35317

[34] Kilbas, A. A.A.; Srivastava, H. M.; Trujillo, J. J., Theory and applications Of Fractinal Differential Equations, Vol. 204, no. 49-52, 2453-2461 (2006)

[35] Liao, K. F.; Li, Y. S.; Wei, T., The identification of the time-dependent source term in time-fractional diffusion-wave equations, East Asian J. Appl. Math., 9, 2, 330-354 (2019) · Zbl 1468.65128

[36] Brezis, H., Functional Analysis, Sobolev Spaces and Partial Differential Equations (2011), Springer: Springer New York · Zbl 1220.46002

[37] Slodička, M.; Šišková, K., An inverse source problem in a semilinear time-fractional diffusion equation, Comput. Math. Appl., 72, 6, 1655-1669 (2016) · Zbl 1365.35217

[38] Fujiwara, D., Concrete characterization of the domains of fractional powers of some elliptic differential operators of the second order, Proc. Japan Acad., 43, 2, 82-86 (1967) · Zbl 0154.16201

[39] Gorenflo, R.; Yamamoto, M., Operator theoretic treatment of linear abel integral equations of first kind, Jpn. J. Ind. Appl. Math., 16, 1, 137-161 (1999) · Zbl 1306.47057

[40] Engl, H. W.; Hanke, M.; Neubauer, A., (Regularization of Inverse Problems. Regularization of Inverse Problems, Mathematics and its Applications, vol. 375 (1996), Kluwer Academic Publishers Group: Kluwer Academic Publishers Group Dordrecht), viii+321 · Zbl 0859.65054

[41] Hubmer, S.; Ramlau, R., Convergence analysis of a two-point gradient method for nonlinear ill-posed problems, Inverse Problems, 33, 9 (2017) · Zbl 1378.65116

[42] Morozov, V. A., Methods for Solving Incorrectly Posed Problems, 102 (1984), Springer-Verlag

[43] Y. Nesterov, A method of solving a convex programming problem with convergence rate \(O (1 / k^2)\), in: Soviet

Mathematics Doklady, 1983. · Zbl 0535.90071

[44] Jin, B. T.; Lazarov, R.; Zhou, Z., Two fully discrete schemes for fractional diffusion and diffusion-wave equations with nonsmooth data, SIAM J. Sci. Comput., 38, 1, A146-A170 (2016) · Zbl 1381.65082