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A moving mesh refinement based optimal accurate uniformly convergent computational method for a parabolic system of boundary layer originated reaction-diffusion problems with arbitrary small diffusion terms. (English) Zbl 07444599

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Summary: In this paper, a system of time dependent boundary layer originated reaction dominated problems with diffusion parameters of different magnitudes, is considered for numerical analysis. The presence of these parameters lead to the boundary layer phenomena. Here, an optimal order uniformly accurate boundary layer adaptive method moving mesh method is proposed. This method is able to capture the layer phenomena without using a priori information of the solution. The problem is discretized by a modified implicit-Euler scheme in time direction. For the present system, adaptive mesh generation is required in space due to the singularly perturbed nature of the problem. For this purpose, a positive error monitor function is used whose equidistribution will move the mesh points towards the boundary layers. Parameter uniform error estimates are derived to show that the convergence rate is optimal with respect to the problem discretization. Numerical experiments strongly verify the theoretical findings and confirm the efficiency and accuracy of the proposed method.

MSC:

65Mxx Numerical methods for partial differential equations, initial value and time-dependent initial-boundary value problems
35B25 Singular perturbations in context of PDEs
35B50 Maximum principles in context of PDEs
35B51 Comparison principles in context of PDEs
35K51 Initial-boundary value problems for second-order parabolic systems
35K57 Reaction-diffusion equations
65L11 Numerical solution of singularly perturbed problems involving ordinary differential equations
76M45 Asymptotic methods, singular perturbations applied to problems in fluid mechanics
65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs
65M15 Error bounds for initial value and initial-boundary value problems involving PDEs
65M50 Mesh generation, refinement, and adaptive methods for the numerical solution of initial value and initial-boundary value problems involving PDEs
65N50 Mesh generation, refinement, and adaptive methods for boundary value problems involving PDEs

Keywords:

adaptive moving mesh; r-refinement method; boundary layer; parabolic system of reaction-diffusion problems; coupled system of PDEs; singularly perturbed problem; modified implicit Euler scheme

Full Text: DOI

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