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Stability bounds of a delay visco-elastic rheological model with substrate friction. (English)

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Summary: Cells and tissues exhibit sustained oscillatory deformations during remodelling, migration or embryogenesis. Although it has been shown that these oscillations correlate with intracellular biochemical signalling, the role of these oscillations is as yet unclear, and whether they may trigger drastic cell reorganisation events or instabilities remains unknown. Here, we present a rheological model that incorporates elastic, viscous and frictional components, and that is able to generate oscillatory response through a delay adaptive process of the rest-length. We analyse its stability as a function of the model parameters and deduce analytical bounds of the stable domain. While increasing values of the delay and remodelling rate render the model unstable, we also show that increasing friction with the substrate destabilises the oscillatory response. This fact was unexpected and still needs to be verified experimentally. Furthermore, we numerically verify that the extension of the model with non-linear deformation measures is able to generate sustained oscillations converging towards a limit cycle. We interpret this sustained regime in terms of non-linear time varying stiffness parameters that alternate between stable and unstable regions of the linear model. We also note that this limit cycle is not present in the linear model. We study the phase diagram and the bifurcations of the non-linear model, based on our conclusions on the linear one. Such dynamic analysis of the delay visco-elastic model in the presence of friction is absent in the literature for both linear and non-linear rheologies. Our work also shows how increasing values of some parameters such as delay and friction decrease its stability, while other parameters such as stiffness stabilise the oscillatory response.

MSC:

34K60 Qualitative investigation and simulation of models involving functional-differential equations
34K20 Stability theory of functional-differential equations
34K13 Periodic solutions to functional-differential equations
74C10 Small-strain, rate-dependent theories of plasticity (including theories of viscoplasticity)
92C37 Cell biology

Keywords:

oscillations; delay differential equations; visco-elasticity; friction; stability; rheology; cells

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