Lachowicz, Mirosław; **Leszczyński, Henryk**; **Topolski, Krzysztof A.**

Approximations of kinetic equations of swarm formation: convergence and exact solutions.

(English) Zbl 07442852
Appl. Math. Comput. 417, Article ID 126778, 16 p. (2022)

Summary: In the present paper we study Euler-type approximations along characteristics for a class of kinetic equations that describe swarm formations in the case when the interactions rate is variable. The proposed numerical schemes preserve essential properties of the kinetic equations and in particular preserve the probabilistic measure and are able to approximate the solution almost to the appearance of blow-ups. The blow-ups are referred here to the self-organization swarm behavior. Moreover we define a class of exact solutions – traveling wave-type equilibrium solutions that we called **TWES**.

MSC:

35Qxx Partial differential equations of mathematical physics and other areas of application
92Dxx Genetics and population dynamics
82Cxx Time-dependent statistical mechanics (dynamic and nonequilibrium)

Keywords:

Euler method; kinetic equations; stability; blow-ups; exact solutions

Full Text: DOI

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