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Kong, Xinlei; Wang, Zhongxin; Wu, Huibin

Variational integrators for forced Lagrangian systems based on the local path fitting technique. (English) Zbl 07442828

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Summary: Variational integrators are particularly suitable for simulation of mechanical systems, where features such as symplecticity and momentum preservation are essential. They also exhibit excellent long-time energy behavior even if external forcing is involved. Motivated by this fact, we present a new approach, that is based on the local path fitting technique, to construct variational integrators for forced mechanical systems. The core technology exploited is to fit the local trajectory as the Lagrange interpolation polynomial by requiring that the forced Euler-Lagrange equations hold at the internal interpolation nodes. This operation also yields the essential terms of the discrete forced Euler-Lagrange equations and consequently formulates the final integrator. This new approach not only avoids numerical quadrature involved in the classical construction, but also significantly improves the precision of the resulting integrator, as illustrated by the given examples.

MSC:

70G75 Variational methods for problems in mechanics
65P10 Numerical methods for Hamiltonian systems including symplectic integrators
92-08 Computational methods for problems pertaining to biology

Keywords:

variational integrator; forced Euler-Lagrange equations; local path fitting; Lagrange interpolation polynomial

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