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On global existence and blowup of solutions of stochastic Keller-Segel type equation. (English) [Zbl 1479.35146](#)

NoDEA, Nonlinear Differ. Equ. Appl. 29, No. 1, Paper No. 3, 29 p. (2022).

Summary: In this paper we consider a stochastic Keller-Segel type equation, perturbed with random noise. We establish that for special types of random perturbations (i.e. in a divergence form), the equation has a global weak solution for small initial data. Furthermore, if the noise is not in a divergence form, we show that the solution has a finite time blowup (with nonzero probability) for any nonzero initial data. The results on the continuous dependence of solutions on the small random perturbations, alongside with the existence of local strong solutions, are also derived in this work.

MSC:

- [35B44](#) Blow-up in context of PDEs
- [35K15](#) Initial value problems for second-order parabolic equations
- [35K59](#) Quasilinear parabolic equations
- [35R60](#) PDEs with randomness, stochastic partial differential equations
- [60H30](#) Applications of stochastic analysis (to PDEs, etc.)
- [65M75](#) Probabilistic methods, particle methods, etc. for initial value and initial-boundary value problems involving PDEs
- [92C17](#) Cell movement (chemotaxis, etc.)

Keywords:

Keller-Segel equation; stochastic partial differential equation; local and global solutions; elliptic-parabolic system

Full Text: [DOI](#) [arXiv](#)

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