Cai, Tianxin; Zhang, Yong

A variety of Euler's sum of powers conjecture. (English) Zbl 07442476

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Summary: We consider a variety of Euler's sum of powers conjecture, i.e., whether the Diophantine system

$$\begin{cases} n = a_1 + a_2 + \cdots + a_{s-1}, \\ a_1 a_2 \cdots a_{s-1}(a_1 + a_2 + \cdots + a_{s-1}) = b^s \end{cases}$$

has positive integer or rational solutions n, b, a_i, $i = 1, 2, \ldots, s-1$, $s \geq 3$. Using the theory of elliptic curves, we prove that it has no positive integer solution for $s = 3$, but there are infinitely many positive integers n such that it has a positive integer solution for $s \geq 4$. As a corollary, for $s \geq 4$ and any positive integer n, the above Diophantine system has a positive rational solution. Meanwhile, we give conditions such that it has infinitely many positive rational solutions for $s \geq 4$ and a fixed positive integer n.

MSC:

11D72 Diophantine equations in many variables
11D41 Higher degree equations; Fermat's equation
11G05 Elliptic curves over global fields

Keywords:

Euler's sum of powers conjecture; elliptic curve; positive integer solution; positive rational solution

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