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Three-dimensional quasi-geostrophic staggered vortex arrays. (English) Zbl 1479.76042

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Summary: We determine and characterise relative equilibria for arrays of point vortices in a three-dimensional quasi-geostrophic flow. The vortices are equally spaced along two horizontal rings whose centre lies on the same vertical axis. An additional vortex may be placed along this vertical axis. Depending on the parameters defining the array, the vortices on the two rings are of equal or opposite sign. We address the linear stability of the point vortex arrays. We find both stable equilibria and unstable equilibria, depending on the geometry of the array. For unstable arrays, the instability may lead to the quasi-regular or to the chaotic motion of the point vortices. The linear stability of the vortex arrays depends on the number of vortices in the array, on the radius ratio between the two rings, on the vertical offset between the rings and on the vertical offset between the rings and the central vortex, when the latter is present. In this case the linear stability also depends on the strength of the central vortex. The non-linear evolution of a selection of unstable cases is presented exhibiting examples of quasi-regular motion and of chaotic motion.

MSC:

76E20 Stability and instability of geophysical and astrophysical flows
76B47 Vortex flows for incompressible inviscid fluids
76U60 Geophysical flows

Keywords:

linear stability anylysis; quasi-geostrophic flow; point vortex dynamics; equilibrium; vortex array

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