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Zhuravlev, Nikolai B.; Rossovskii, Leonid E.

Spectral radius formula for a parametric family of functional operators. (English)

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Summary: The conditions for the unique solvability of the boundary-value problem for a functional differential equation with shifted and compressed arguments are expressed via the spectral radius formula for the corresponding class of functional operators. The use of this formula involves calculation of certain type limits, which, even in the simplest cases, exhibit an amazing "chaotic" dependence on the compression ratio. For example, it turns out that the spectral radius of the operator

$$L_2(\mathbb{R}^n) \ni u(x) \mapsto u(p^{-1}x+h) - u(p^{-1}x-h) \in L_2(\mathbb{R}^n), \quad p>1, \quad h \in \mathbb{R}^n,$$

is equal to $2p^{n/2}$ for transcendental values of p, and depends on the coefficients of the minimal polynomial for p in the case where p is an algebraic number. In this paper, we study this dependence. The starting point is the well-known statement that, given a velocity vector with rationally independent coordinates, the corresponding linear flow is minimal on the torus, i.e., the trajectory of each point is everywhere dense on the torus. We prove a version of this statement that helps to control the behavior of trajectories also in the case of rationally dependent velocities. Upper and lower bounds for the spectral radius are obtained for various cases of the coefficients of the minimal polynomial for p. The main result of the paper is the exact formula of the spectral radius for rational (and roots of any degree of rational) values of p.

MSC:

35J25 Boundary value problems for second-order elliptic equations
39A13 Difference equations, scaling (q-differences)

Keywords:

elliptic functional differential equation; differential-difference equation

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References:

[1] Derfel, G. A.; Molchanov, S. A., Spectral Methods in the Theory of Functional-Differential Equations, Math. Notes, 47, 3-4, 254-260 (1990) · Zbl 0760.34055 · doi:10.1007/BF01138504

[2] Iserles, A., On Neutral Functional-Differential Equation with Proportional Delays, J. Math. Anal. Appl., 207, 1, 73-95 (1997) · Zbl 0873.34066 · doi:10.1006/jmaa.1997.5262

[3] Kato, T.; McLeod, J. B., The Functional-Differential Equation \(y^{\prime}(x)=ay(\lambda x)+by(x)\), Bull. Amer. Math. Soc., 77, 6, 891-937 (1971) · Zbl 0236.34064 · doi:10.1090/S0002-9904-1971-12805-7

[4] Katok, A.; Hasselblatt, B., Introduction to the Modern Theory of Dynamical Systems (1995), Cambridge: Cambridge Univ. Press, Cambridge · Zbl 0878.58020 · doi:10.1017/CBO9780511809187

[5] Rossovskii, L. E., Elliptic Functional Differential Equations with Contractions and Extensions of Independent Variables of the Unknown Function, J. Math. Sci., 223, 4, 351-493 (2017) · Zbl 1442.35495 · doi:10.1007/s10958-017-3360-1

[6] Rossovskii, L. E., Elliptic Functional Differential Equations with Incommensurable Contractions, Math. Model. Nat. Phenom., 12, 6, 226-239 (2017) · Zbl 1386.35066 · doi:10.1051/mmnp/2017075

[7] Rossovskii, L. E.; Tovsultanov, A. A., Elliptic Functional Differential Equation with Affine Transformations, J. Math. Anal. Appl., 480, 2 (2019) · Zbl 1431.35029 · doi:10.1016/j.jmaa.2019.123403

[8] Skubachevskii, A. L., Elliptic Functional-Differential Equations and Applications (1997), Basel: Birkhäuser, Basel · Zbl 0946.35113

[9] Skubachevskii, A. L., Boundary Value Problems for Elliptic Functional-Differential Equations and Their Applications, Russian Math. Surveys, 71, 5, 801-906 (2016) · Zbl 1364.35413 · doi:10.1070/RM9739