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Second-order BDF ADI Galerkin finite element method for the evolutionary equation with a nonlocal term in three-dimensional space. (English) [Zbl 07441568](#)

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Summary: In this work, we propose and analyze a new method for the solution of the three-dimensional evolutionary equation with a nonlocal term. Then the method combines Galerkin finite element methods (FEMs) for the spatial discretization with an alternating direction implicit (ADI) algorithm based on the second-order backward differentiation formula (BDF2), where the Riemann-Liouville (R-L) integral term is approximated via second-order convolution quadrature (CQ) rule. The L^2 -norm stability and convergence are proved. Numerical results confirm the predicted space-time convergence rates.

MSC:

65M60 Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs

65M12 Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs

Keywords:

three-dimensional nonlocal evolution equation; BDF2 ADI Galerkin method; second-order convolution quadrature rule; stability and convergence; numerical experiments

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