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Linearly implicit variable step-size BDF schemes with Fourier pseudospectral approximation for incompressible Navier-Stokes equations. (English) Zbl 07441563

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Summary: In this paper, linearly implicit backward differentiation formulas with variable step-sizes are proposed to solve numerically the two-dimensional incompressible Navier-Stokes equations (formulated in vorticity-stream function). With Fourier pseudospectral methods for spatial discretization, the diffusion term is discretized implicitly and the nonlinear convection term is treated by a combination of implicit and explicit discretizations. As a result, only linear solvers are needed at each time step to achieve the desired temporal accuracy. With the help of a priori assumption and aliasing error control techniques, the error estimates for one-step and two-step backward differentiation formulas are established in several norms under appropriate step-size constraints. Compared with the numerical results of implicit-explicit (the nonlinear convection term is treated explicitly) BDF2 method and fully implicit Crank-Nicolson method, it demonstrates that the proposed linearly implicit variable step-size BDF2 method is effective and robust.

MSC:

65M70 Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs
65L06 Multistep, Runge-Kutta and extrapolation methods for ordinary differential equations
65M12 Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs
76D05 Navier-Stokes equations for incompressible viscous fluids

Keywords:

incompressible Navier-Stokes equations; linearly implicit methods; two-step backward differentiation formula; linearly implicit Euler scheme; Fourier pseudospectral approximation; stability and convergence

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