Qiao, Leijie; Xu, Da; Qiu, Wenlin

The formally second-order BDF ADI difference/compact difference scheme for the nonlocal evolution problem in three-dimensional space. (English) Zbl 07441561

Appl. Numer. Math. 172, 359-381 (2022)

Summary: This work formulates two kinds of alternating direction implicit (ADI) schemes for the parabolic-type three-dimensional evolution equation with a weakly singular kernel. The second-order backward differentiation formula (BDF2) and the second-order convolution quadrature (CQ) technique are applied to the discretization of the time derivative and the Riemann-Liouville (R-L) integral, respectively. Then, the fully-discrete BDF2 difference scheme and BDF2 compact difference scheme are constructed via the general centered difference and compact difference method, respectively. Meanwhile, the ADI algorithms are designed reasonably for two schemes to reduce the computational cost. The stability and convergence of two ADI schemes are derived via the energy method. Finally, several numerical examples are provided and tested to validate the theoretical analysis.

MSC:

65R20	Numerical methods for integral equations
45K05	Integro-partial differential equations
35R11	Fractional partial differential equations
65M06	Finite difference methods for initial value and initial-boundary value problems involving PDEs
65M12	Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs

Keywords:

three-dimensional parabolic-type evolution equation; ADI difference/compact difference schemes; second-order convolution quadrature; stability and convergence; numerical experiments

Software:

Algorithm 986

Full Text: DOI

References:

[1] Bateman, H., Higher Transcendental Functions, vol. 1 (1953), McGraw-Hill: McGraw-Hill New York

[2] Cen, D.; Wang, Z.; Mo, Y., Second order difference schemes for time-fractional KdV-Burgers' equation with initial singularity, Appl. Math. Lett., 112, Article 106829 pp. (2021) · Zbl 1453.65210

[3] Chen, C.; Thomée, V.; Wahlbin, L. B., Finite element approximation of a parabolic integro-differential equation with a weakly singular kernel, Math. Comput., 58, 587-602 (1992) · Zbl 0766.65120

[4] Chen, H.; Xu, D.; Peng, Y., A second order BDF alternating direction implicit difference scheme for the two-dimensional fractional evolution equation, Appl. Math. Model., 41, 54-67 (2017) · Zbl 1443.65439

[5] Chen, H.; Xu, D.; Cao, J.; Zhou, J., A formally second order BDF ADI difference scheme for the three-dimensional time-fractional heat equation, Int. J. Comput. Math., 97, 1100-1117 (2020)

[6] Chung, S.; Park, M., Spectral analysis for hyperbolic integro-differential equations with a weakly singular kernel, J. Korean Soc. Ind. Appl. Math., 2, 31-40 (1998)

[7] Du, R.; Sun, Z., A fast temporal second-order compact ADI scheme for time fractional mixed diffusion-wave equations, East Asian J. Appl. Math., 11, 647-673 (2021)

[8] Khebchareon, M.; Pani, A. K.; Fairweather, G., Alternating direction implicit Galerkin methods for an evolution equation with a positive-type memory term, J. Sci. Comput., 65, 1166-1188 (2015) · Zbl 1328.65209

[9] Kim, C.; Choi, U., Spectral collocation methods for a partial integro-differential equation with a weakly singular kernel, J. Aust. Math. Soc. Ser. B, 39, 408-430 (1998) · Zbl 0899.65080

[10] Kumar, N.; Mehra, M., Legendre, wavelet collocation method for fractional optimal control problems with fractional Bolza cost, Numer. Methods Partial Differ. Equ., 37, 1693-1724 (2021)

[11] Kumar, N.; Mehra, M., Collocation method for solving non-linear fractional optimal control problems by using Hermite scaling

function with error estimates, Optim. Control Appl. Methods, 42, 417-444 (2021) · Zbl 1468.49032

[12] Larsson, S.; Thomée, V.; Wahlbin, B., Numerical solution of parabolic integro-differential equations by the discontinuous Galerkin methods, Math. Comput., 67, 45-71 (1998) · Zbl 0896.65090

[13] Lin, Y.; Thomée, V.; Wahlbin, B., Ritz-Volterra projections to finite element spaces and applications to integrodifferential and related equations, SIAM J. Numer. Anal., 2, 1047-1070 (1991) · Zbl 0728.65117

[14] Lopez-Marcos, J., A difference scheme for a nonlinear partial integro-differential equation, SIAM J. Numer. Anal., 27, 20-31 (1990) · Zbl 0693.65097

[15] Lubich, C., Discretized fractional calculus, SIAM J. Math. Anal., 17, 704-719 (1986) · Zbl 0624.65015

[16] Lubich, C., Convolution quadrature and discretized operational calculus, I, Numer. Math., 52, 129-145 (1988) · Zbl 0637.65016

[17] Mclean, W.; Thomée, V., Numerical solution of an evolution equation with a positive type memory term, J. Aust. Math. Soc. Ser. B, 35, 23-70 (1993) · Zbl 0791.65105

[18] Mehandiratta, V.; Mehra, M.; Leugering, G., Fractional optimal control problems on a star graph: optimality system and numerical solution, Math. Control Relat. Fields, 11, 189-209 (2021) · Zbl 1460.34017

[19] Mehandiratta, V.; Mehra, M.; Leugering, G., Existence results and stability analysis for a nonlinear fractional boundary value problem on a circular ring with an attached edge: a study of fractional calculus on metric graph, Netw. Heterog. Media, 16, 155-185 (2021) · Zbl 1465.34010

[20] Mehra, M.; Patel, K., Algorithm 986: a suite of compact finite difference schemes, ACM Trans. Math. Softw., 44, 1-31 (2017) · Zbl 06920086

[21] Mustapha, K.; Mustapha, H., A second-order accurate numerical method for a semilinear integro-differential equation with a weakly singular kernel, IMA J. Numer. Anal., 30, 555-578 (2009) · Zbl 1193.65225

[22] Patel, K.; Mehra, M., Fourth order compact scheme for space fractional advection-diffusion reaction equations with variable coefficients, J. Comput. Appl. Math., 380, Article 112963 pp. (2020) · Zbl 1440.65097

[23] Podlubny, I., Fractional Differential Equations (1999), Academic Press: Academic Press San Diego · Zbl 0918.34010

[24] Qiao, L.; Xu, D., Compact ADI scheme for integro-differential equations of parabolic type, J. Sci. Comput., 76, 565-582 (2018) · Zbl 1445.65050

[25] Qiao, L.; Xu, D.; Wang, Z., An ADI difference scheme based on fractional trapezoidal rule for fractional integro-differential equation with a weakly singular kernel, Appl. Math. Comput., 354, 103-114 (2019) · Zbl 1429.65196

[26] Qiao, L.; Wang, Z.; Xu, D., An alternating direction implicit orthogonal spline collocation method for the two-dimensional multi-term time fractional integro-differential equation, Appl. Numer. Math., 151, 199-212 (2020) · Zbl 1439.65229

[27] Serna, S., A numerical method for a partial integro-differential equation, SIAM J. Numer. Anal., 25, 319-327 (1988) · Zbl 0643.65098

[28] Singh, A.; Mehra, M., Wavelet collocation method based on Legendre polynomials and its application in solving the stochastic fractional integro-differential equations, J. Comput. Sci., 51, Article 101342 pp. (2021)

[29] Sloan, H.; Thomée, V., Time discretization of an integro-differential equation of parabolic type, SIAM J. Numer. Anal., 23, 1052-1061 (1986) · Zbl 0608.65096

[30] Sun, Z., The Method of Order Reduction and Its Application to the Numerical Solutions of Partial Differential Equations (2009), Science Press: Science Press Beijing

[31] Tang, T., A finite difference scheme for partial integro-differential equations with a weakly singular kernel, Appl. Numer. Math., 11, 309-319 (1993) · Zbl 0768.65093

[32] Uchaikin, V., Fractional Derivatives for Physicists and Engineers: vol. I Background and Theory, vol. II Applications (2013), Springer · Zbl 1312.26002

[33] Vong, S.; Lyu, P.; Chen, X.; Lei, S., High order finite difference method for time-space fractional differential equations with Caputo and Riemann-Liouville derivatives, Numer. Algorithms, 72, 195-210 (2015) · Zbl 1382.65259

[34] Wang, Z.; Vong, S., A high-order exponential ADI scheme for two dimensional time fractional convection-diffusion equations, Comput. Math. Appl., 68, 185-196 (2014) · Zbl 1369.65105

[35] Wang, Z.; Cen, D.; Mo, Y., Sharp error estimate of a compact L1-ADI scheme for the two-dimensional time-fractional integro-differential equation with singular kernels, Appl. Numer. Math., 159, 190-203 (2021) · Zbl 1459.65160

[36] Wazwaz, A., Linear and Nonlinear Integral Equations: Methods and Applications (2011), Springer

[37] Xu, D., On the discretization in time for a parabolic integro-differential equation with a weakly singular kernel, I: smooth initial data, Appl. Math. Comput., 58, 1-27 (1993) · Zbl 0782.65160

[38] Xu, D., Finite element methods for the nonlinear integro-differential equations, Appl. Math. Comput., 58, 241-273 (1993) · Zbl 0786.65127

[39] Xu, D., The global behavior of time discretization for an abstract Volterra equation in Hilbert space, Calcolo, 34, 71-104 (1997) · Zbl 0913.65138

[40] Xu, D., The uniform \(L^2\) behavior for time discretization of an evolution equation, Acta Math. Sin. Engl. Ser., 19, 127-140 (2003) · Zbl 1029.65143

[41] Xu, D.; Qiu, W.; Guo, J., A compact finite difference scheme for the fourth-order time-fractional integro-differential equation with a weakly singular kernel, Numer. Methods Partial Differ. Equ., 36, 439-458 (2020)

[42] Yanik, E.; Fairweather, G., Finite element methods for parabolic and hyperbolic partial integro-differential equations, Non-

linear Anal., 12, 785-809 (1988) · Zbl 0657.65142

[43] Qiao, L.; Qiu, W.; Xu, D., A second-order ADI difference scheme based on non-uniform meshes for the three-dimensional nonlocal evolution problem, Comput. Math. Appl., 102, 137-145 (2021) · Zbl 07419184

[44] Yang, X.; Qiu, W.; Zhang, H.; Tang, L., An efficient alternating direction implicit finite difference scheme for the three-dimensional time-fractional telegraph equation, Comput. Math. Appl., 102, 233-247 (2021) · Zbl 07419191