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MSC:

35A23 Inequalities applied to PDEs involving derivatives, differential and integral operators, or integrals
35A15 Variational methods applied to PDEs
35Q41 Time-dependent Schrödinger equations and Dirac equations
35Q55 NLS equations (nonlinear Schrödinger equations)
35Q60 PDEs in connection with optics and electromagnetic theory
49J20 Existence theories for optimal control problems involving partial differential equations
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