Mohamed, Kamel; Abdelrahman, Mahmoud A. E.

The modified Rusanov scheme for solving the ultra-relativistic Euler equations. (English)

Zbl 07440254
Eur. J. Mech., B, Fluids 90, 89-98 (2021)

Summary: We consider the ultra-relativistic Euler equations for an ideal gas, which are described in terms of the pressure p and the spatial part $\mathbf{u} \in \mathbb{R}^3$ of the dimensionless four-velocity. We also numerically investigate the ultra-relativistic Euler equations using the modified Rusanov scheme compared with classical Rusanov scheme. Our scheme consists of predictor and corrector stage. The predictor stage contains a parameter of control $(\alpha^n_{i+\frac{1}{2}})$ is responsible for the numerical diffusion of this scheme. In order to control this parameter, we use a strategy depending on limiter theory and using Riemann invariants. The corrector stage recovers the balance conservation equation. The scheme can compute the numerical flux corresponding to the real state of solution without relying on Riemann problem solvers. The numerical results show the high resolution of the proposed finite volume scheme (modified Rusanov) and confirm its capability to provide accurate simulations for the ultra-relativistic Euler equations under regimes with strong shocks and rarefactions.

MSC:

65-XX Numerical analysis
35L45 Initial value problems for first-order hyperbolic systems
35L65 Hyperbolic conservation laws
35L67 Shocks and singularities for hyperbolic equations
65M08 Finite volume methods for initial value and initial-boundary value problems involving PDEs
76Y05 Quantum hydrodynamics and relativistic hydrodynamics
76-XX Fluid mechanics

Keywords:

relativistic Euler equations; entropy shocks; rarefaction waves; Riemann invariants; Riemann solutions; finite volume scheme; modified Rusanov scheme

Full Text: DOI

References:

[1] Del Zanna, L.; Bucciantini, N., An efficient shock-capturing central-type scheme for multidimensional relativistic flows, I. Hydrodyn. Astron. Astrophys., 390, 1177-1186 (2002) · Zbl 1209.76022

[2] Donat, R.; Marquina, A., Capturing shock reflections: An improved flux formula, J. Comput. Phys., 125, 42-58 (1996) · Zbl 0847.76049

[3] Eulderink, F.; Mellema, G., General relativistic hydrodynamics with a Roe solver, Astron. Astrophys. Suppl., 110, 587-623 (1995)

[4] Lucas-Serrano, A.; Font, J. A.; Ibez, J. M.; Mart, J. M., Assessment of a high-resolution central scheme for the solution of the relativistic hydrodynamics equations, I. Hydrodyna. Astron. Astrophys., 428, 703-715 (2004) · Zbl 1065.85009

[5] Schneider, V.; Katscher, U.; Rischke, D. H.; Waldhauser, B.; Maruhn, J. A.; Munz, C.-D., New algorithms for ultra-relativistic numerical hydrodynamics, J. Comput. Phys., 105, 92-107 (1993) · Zbl 0779.76062

[6] Kunik, M.; Qamar, S.; Warnecke, G., Kinetic schemes for the ultra-relativistic Euler equations, J. Comput. Phys., 187, 572-596 (2003) · Zbl 1061.76068

[7] Kunik, M.; Qamar, S.; Warnecke, G., BGK-type kinetic flux vector splitting schemes for ultra-relativistic Euler equations, SIAM J. Sci. Comput., 26, 196-223 (2004) · Zbl 1071.82049

[8] Kunik, M.; Qamar, S.; Warnecke, G., Kinetic schemes for the relativistic gas dynamics, Numer. Math., 97, 159-191 (2004) · Zbl 1098.76056

[9] Abdelrahman, M. A.E.; Kunik, M., A new front tracking scheme for the ultra-relativistic Euler equations, J. Comput. Phys., 275, 213-235 (2014) · Zbl 1349.76579

[10] Mohamed, K., Simulation numérique en volume finis, de problémes d'écoulements multidimensionnels raides, par un schéma de fluxá deux pas, (Dissertation (2005), University of Paris), 13

[11] Toro, E. F., Riemann Solvers and Numerical Methods for Fluid Dynamics (1999), Springer: Springer Berlin · Zbl 0923.76004

[12] Abdelrahman, M. A.E., On the shallow water equations, Z. Nat.forsch., 72, 9a, 873-879 (2017)

[13] Glimm, J., Solutions in the large for nonlinear hyperbolic systems of equations, Comm. Pure. Appl. Math., 18, 697-715 (1995) · Zbl 0141.28902

[14] Chen, G. Q.; Li, Y. C., Relativistic Euler equations for isentropic fluids: stability of Riemann solutions with large oscillation, Z. Angew. Math. Phys., 55, 903-926 (2004) · Zbl 1068.35099

[15] Wissman, B. D., Global solutions to the ultra-relativistic Euler equations, Comm. Math. Phys., 306, 831-851 (2011) · Zbl 1231.35160

[16] Abdelrahman, M. A.E.; Kunik, M., The interaction of waves for the ultra-relativistic Euler equations, J. Math. Anal. Appl., 409, 1140-1158 (2014) · Zbl 1309.35062

[17] Abdelrahman, M. A.E.; Kunik, M., The ultra-relativistic Euler equations, Math. Methods Appl. Sci., 38, 1247-1264 (2015) · Zbl 1319.35170

[18] Abdelrahman, M. A.E., Global solutions for the ultra-relativistic Euler equations, Nonlinear Anal., 155, 140-162 (2017) · Zbl 1369.35046

[19] Abdelrahman, M. A.E., Numerical investigation of the wave-front tracking algorithm for the full ultra-relativistic Euler equations, Int. J. Nonlinear Sci. Numer. Simul., 19, 2, 223-229 (2018) · Zbl 1401.35212

[20] Chen, J., Conservation laws for the relativistic p-system, Comm. Partial Differential Equations, 20, 1602-1646 (1995) · Zbl 0877.35096

[21] Kunik, M., Selected Initial and Boundary Value Problems for Hyperbolic Systems and Kinetic Equations (2005), Otto-von-Guericke University, (Habilitation thesis)

[22] Smoller, J.; B. Temple, M., Global solutions of the relativistic Euler equations, Comm. Math. Phys., 156, 67-99 (1993) · Zbl 0780.76085

[23] Abdelrahman, M. A.E.; El-Shafei, M. E.; Alsammarraie, H. A., Riemann invariants for the ultra-relativistic Euler equations, IJCTA., 5, 6, 3057-3066 (2014)

[24] Mohamed, K.; Seaid, M.; Zahri, M., A finite volume method for scalar conservation laws with stochastic time-space dependent flux function, J. Comput. Appl. Math., 237, 614-632 (2013) · Zbl 1252.65152

[25] F. Benkhaldoun, K. Mohamed, M. Seaid, A Generalized Rusanov method for Saint-Venant Equations with Variable Horizontal Density. in: FVCA international symposium, Prague; June, 2011, vol. 6-10, pp.96-112. · Zbl 1246.76065

[26] Mohamed, K.; Benkhaldoun, F., A modified Rusanov scheme for shallow water equations with topography and two phase flows, Eur. Phys. J. Plus, 131, 207 (2016)

[27] Mohamed, K., A finite volume method for numerical simulation of shallow water models with porosity, Comput. Fluids., 104, 9-19 (2014) · Zbl 1391.76431

[28] Mohamed, K.; Seadawy, A. R., Finite volume scheme for numerical simulation of the sediment transport model, Internat. J. Modern Phys. B, 33, 24, Article 1950283 pp. (2019), 24 pages · Zbl 1428.76123

[29] LeVeque, R. J., Numerical methods for conservation laws (1992), Birkhäuser Verlag Basel: Birkhäuser Verlag Basel Switzerland · Zbl 0847.65053

[30] Sweby, P. K., High resolution schemes using flux limiters for hyperbolic conservation laws, SIAM J. Numer. Anal., 21, 995-1011 (1984) · Zbl 0565.65048

[31] Van Leer, B., Towards the ultimate conservative difference schemes V. A second-order Ssequal to Godunov's method, J. Comput. Phys., 32, 101-136 (1979) · Zbl 1364.65223