Ebanks, B. R.; Heuvers, K. J.; Ng, C. T.

On Cauchy differences of all orders. (English) Zbl 0744.39003

Aequationes Math. 42, No. 2-3, 137-153 (1991).

The Cauchy difference of order 1 of a mapping f is $f(x+y)-f(x)-f(y)$. *B. Jessen*, *J. Karpf* and *A. Thorup* [Math. Scand. 22, 257–265 (1968; Zbl 0183.04004)] characterized as a Cauchy difference of order 1 the two place function $S: G^2 \to X$ where G is an abelian group and X a divisible abelian group. The characterization is unique up to an additive function.

The Cauchy difference of order 2 is $f(x+y+z)-f(x+y)-f(x+z)-f(y+z)+f(x)+f(y)+f(z)$. The authors characterize three place functions $\Sigma: G^3 \to X$, for G an abelian group and X a rational vector space, as Cauchy differences of order 2 unique up to a generalized polynomial of degree 2. They then show this result generalizes naturally but not trivially to order $n>2$.

In the statement of the uniqueness conditions of the generalization, the expression $\frac{1}{n!}\sigma S_{12\dots n}$ on line 2 of p. 145 should read $\frac{1}{n!}\delta S_{12\dots n}$.

Reviewer: Mark A.Taylor (Wolfville)

MSC:

39B52	Functional equations for functions with more general domains and/or ranges
39A70	Difference operators

Keywords:

Cauchy difference; divisible abelian group; additive function; generalized polynomial

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