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A compactness result for inhomogeneous nonlinear Schrödinger equations. (English)

Zbl 1479.35777

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Summary: We establish a compactness property of the difference between nonlinear and linear operators (or the Duhamel operator) related to the inhomogeneous nonlinear Schrödinger equation. The proof is based on a refined profile decomposition for the equation. More precisely, we prove that any sequence $(\phi_n)_n$ of H^1 -functions which converges weakly in H^1 to a function ϕ , the corresponding solutions with initial data ϕ_n can be decomposed (up to a remainder term) as a sum of the corresponding solution with initial data ϕ and solutions to the linear equation.

MSC:

35Q55 NLS equations (nonlinear Schrödinger equations)

35Q41 Time-dependent Schrödinger equations and Dirac equations

35B44 Blow-up in context of PDEs

35A01 Existence problems for PDEs: global existence, local existence, non-existence

35A02 Uniqueness problems for PDEs: global uniqueness, local uniqueness, non-uniqueness

78A60 Lasers, masers, optical bistability, nonlinear optics

82D10 Statistical mechanics of plasmas

Keywords:

inhomogeneous nonlinear Schrödinger equation; compactness property; linear profile decomposition; nonlinear profile decomposition

Full Text: DOI

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