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Fractional sum and fractional difference on non-uniform lattices and analogue of Euler and Cauchy beta formulas. (English) Zbl 07439146

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Summary: As is well known, the definitions of fractional sum and fractional difference of $f(z)$ on non-uniform lattices $x(z) = c_1z^2 + c_2z + c_3$ or $x(z) = c_1q^z + c_2q^{-z} + c_3$ are more difficult and complicated. In this article, for the first time we propose the definitions of the fractional sum and fractional difference on non-uniform lattices by two different ways. The analogue of Euler's Beta formula, Cauchy' Beta formula on non-uniform lattices are established, and some fundamental theorems of fractional calculas, the solution of the generalized Abel equation on non-uniform lattices are obtained etc.

MSC:

39A13 Difference equations, scaling (q-differences)
33C45 Orthogonal polynomials and functions of hypergeometric type (Jacobi, Laguerre, Hermite, Askey scheme, etc.)
33D45 Basic orthogonal polynomials and functions (Askey-Wilson polynomials, etc.)
26A33 Fractional derivatives and integrals
34K37 Functional-differential equations with fractional derivatives

Keywords:

difference equation of hypergeometric type; non-uniform lattice; fractional sum; fractional difference; special functions; Euler's beta formula; Cauchy' beta formula

Full Text: DOI

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