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Planar vortices in a bounded domain with a hole. (English) Zbl 1478.49004

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Summary: In this paper, we consider the inviscid, incompressible planar flows in a bounded domain with a hole and construct stationary classical solutions with single vortex core, which is closed to the hole. This is carried out by constructing solutions to the following semilinear elliptic problem

$$\begin{cases} -\Delta\psi = \lambda(\psi - \frac{\kappa}{4\pi}\ln\lambda)_+^p & \text{in } \Omega, \\ \psi = \rho_\lambda, & \text{on } \partial O_0 \\ \psi = 0, & \text{on } \partial\Omega_0 \end{cases} \tag{1}$$

where $p > 1$, κ is a positive constant, ρ_λ is a constant, depending on λ, $\Omega = \Omega_0 \setminus \bar{O}_0$ and Ω_0, O_0 are two planar bounded simply-connected domains. We show that under the assumption $(\ln\lambda)^\sigma \leq \rho_\lambda \leq (\ln\lambda)^{1-\sigma}$ for some $\sigma > 0$ small, (1) has a solution ψ_λ, whose vorticity set $\{y \in \Omega : \psi(y) - \kappa + \rho_\lambda\eta(y) > 0\}$ shrinks to the boundary of the hole as $\lambda \to +\infty$.

MSC:

49J20 Existence theories for optimal control problems involving partial differential equations

49S05 Variational principles of physics

76B47 Vortex flows for incompressible inviscid fluids

35J61 Semilinear elliptic equations

Keywords:

Euler flow; semilinear elliptic equation; variational method; free boundary problem; reduction

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