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**Well-posedness results and blow-up for a semi-linear time fractional diffusion equation with variable coefficients.** (English) [Zbl 1478.35218](#)

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**Summary:** The semi-linear problem of a fractional diffusion equation with the Caputo-like counterpart of a hyper-Bessel differential is considered. The results on existence, uniqueness and regularity estimates (local well-posedness) of the solutions are established in the case of linear source and the source functions that satisfy the globally Lipschitz conditions. Moreover, we prove that the problem exists a unique positive solution. In addition, the unique continuation of solutions and a finite-time blow-up are proposed with the reaction terms are logarithmic functions.

**MSC:**

- [35R11](#) Fractional partial differential equations
- [26A33](#) Fractional derivatives and integrals
- [35K15](#) Initial value problems for second-order parabolic equations
- [35B40](#) Asymptotic behavior of solutions to PDEs
- [35B44](#) Blow-up in context of PDEs
- [33E12](#) Mittag-Leffler functions and generalizations
- [44A20](#) Integral transforms of special functions

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**Keywords:**

fractional diffusion equation; hyper-Bessel operators; generalized fractional calculus

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