Kulaitis, Gytis; **Munk, Axel**; **Werner, Frank**

What is resolution? A statistical minimax testing perspective on superresolution microscopy. (English) Zbl 1483.62117

Ann. Stat. 49, No. 4, 2292-2312 (2021).

The authors suggest a statistical description, which allows one to understand both the effect of the experimental setup and the random nature of photon counts on the resulting resolution. On the one hand, for Poisson measurements, they get linear dependence of the (minimax) detection boundary on the full width at half maximum (FWHM) and on the other hand, for a homogeneous Gaussian model the dependence of resolution is nonlinear. They conclude that at small physical scales modeling by homogeneous Gaussians is inadequate. In comparison to the Poisson model, that seems to provide a statistically sound description of resolution at the nanoscale. Their theory seems to be also applicable to other imaging setups.

Reviewer: Makrina Agaoglou (Bristol)

MSC:

62H35 Image analysis in multivariate analysis
62C20 Minimax procedures in statistical decision theory
62F03 Parametric hypothesis testing
62P30 Applications of statistics in engineering and industry; control charts
94A08 Image processing (compression, reconstruction, etc.) in information and communication theory

Keywords:

(super)resolution; detection boundary; equivalence of experiments; microscopy; minimax; nanoscopy

Full Text: DOI arXiv Link

References:

[1] Abbe, E. (1873). Beiträge zur Theorie des Mikroskops und der mikroskopischen Wahrnehmung. \textit{Arch. Mikrosk. Anat.} 9 413-418.· doi:10.1007/BF02956173

[2] Acuña, C. O. and Horowitz, J. (1997). A statistical approach to the resolution of point sources. \textit{J. Appl. Stat.} 24 421-435.· doi:10.1080/02664769723620

[3] Airy, G. B. (1835). On the diffraction of an object-glass with circular aperture. \textit{Trans. Camb. Philos. Soc.} 5 283-291.

[4] Ash, E. A. and Nicholls, G. (1972). Super-resolution aperture scanning microscope. \textit{Nature} 237 510-512.· doi:10.1038/237510a0

[5] Aspelmeier, T., Egner, A. and Munk, A. (2015). Modern statistical challenges in high-resolution fluorescence microscopy. \textit{Annu. Rev. Stat. Appl.} 2 163-202.· doi:10.1007/s00344-014-9442-0

[6] Banterle, N., Bui, K. H., Lemke, E. A. and Beck, M. (2013). Fourier ring correlation as a resolution criterion for super-resolution microscopy. \textit{J. Struct. Biol.} 183 363-367.· doi:10.1016/j.jsb.2013.05.004

[7] Bertero, M., Boccacci, P., Desiderà, G. and Vicidomini, G. (2009). Image deblurring with Poisson data: From cells to galaxies. \textit{Inverse Probl.} 25 123006. · Zbl 1186.85001 · doi:10.1088/0266-5611/25/12/123006

[8] Betzig, E., Davidson, M. W., Patterson, G. H., Lippincott-Schwartz, J., Sougrat, R., Lindwasser, O. W., Olenych, S., Bonifacino, J. S. and Hess, H. F. (2006). Imaging intracellular fluorescent proteins at nanometer resolution. \textit{Science} 313 1642-1645.

[9] Born, M. and Wolf, E. (1999). \textit{Principles of Optics}: \textit{Electromagnetic Theory of Propagation, Interference and Diffraction of Light}, 7th ed. Cambridge Univ. Press, Cambridge.

[10] Candès, E. J. and Fernandez-Granda, C. (2013). Super-resolution from noisy data. \textit{J. Fourier Anal. Appl.} 19 1229-1254. · Zbl 1312.94015 · doi:10.1007/s00041-013-9292-3

[11] Candès, E. J. and Fernandez-Granda, C. (2014). Towards a mathematical theory of super-resolution. \textit{Comm. Pure Appl. Math.} 67 906-956. · Zbl 1350.94011

[12] Courjon, D. (2003). \textit{Near-Field Microscopy and Near-Field Optics}. Imperial College Press, London.

[13] Cremer, C. and Masters, B. R. (2013). Resolution enhancement techniques in microscopy. \textit{Eur. Phys. J. H} 38 281-344.· doi:10.1140/epjh/e2012-20060-1

[14] den Dekker, A. J. and van den Bos, A. (1997). Resolution: A survey. \textit{J. Opt. Soc. Amer. A} 14 547-557.· doi:10.1364/JOSAA.14.000547

[15] Donoho, D. L. (1992). Superresolution via sparsity constraints. \textit{SIAM J. Math. Anal.} 23 1309-1331. · Zbl 0769.42007 · doi:10.1137/0523074

[16] Du, C. and Kou, S. C. (2020). Statistical methodology in single-molecule experiments. \textit{Statist. Sci.} 35 75-91. · Zbl 07292497 · doi:10.1214/19-STS752

[17] Dürig, U., Pohl, D. W. and Rohner, F. (1986). Near-field optical-scanning microscopy. \textit{J. Appl. Phys.} 59 3318-3327.· doi:10.1063/1.336848

[18] Egner, A., Geisler, C. and Siegmund, R. (2020). Basic knowledge in STED nanoscopy. In \textit{Nanoscale Photonic Imaging} (T. Salditt, A. Egner and R. Luke, eds.) Springer, Berlin.

[19] Egner, A., Geisler, C., Von Middendorff, C., Bock, H., Wenzel, D., Medda, R., Andresen, M., Stiel, A. C., Jakobs, S. et al. (2007). Fluorescence nanoscopy in whole cells by asynchronous localization of photoswitching emitters. \textit{Biophys. J.} 93 3285-3290.· doi:10.1529/biophysj.107.112201

[20] Ehrenberg, M. (2014). Super-resolved fluorescence microscopy. Scientific Background on Nobel Prize in Chemistry.

[21] Fernandez-Granda, C. (2015). Super-resolution of point sources via convex programming. In 2015 \textit{IEEE} 6\textit{th International Workshop on Computational Advances in Multi-Sensor Adaptive Processing, CAMSAP} 2015 41-44.

[22] Ferreira Da Costa, M. and Chi, Y. (2020). On the stable resolution limit of total variation regularization for spike deconvolution. \textit{IEEE Trans. Inf. Theory} 66 7237-7252. · Zbl 1453.94025

[23] Goodman, J. W. (1985). \textit{Statistical Optics}. Wiley-Interscience, New York.

[24] Harris, J. L. (1964). Resolving power and decision theory. \textit{J. Opt. Soc. Amer.} 54 606-611.· doi:10.1364/JOSA.54.000606

[25] Heilemann, M., Van De Linde, S., Schüttpelz, M., Kasper, R., Seefeldt, B., Mukherjee, A., Tinnefeld, P. and Sauer, M. (2008). Subdiffraction-resolution fluorescence imaging with conventional fluorescent probes. \textit{Angew. Chem.} 47 6172-6176.· doi:10.1002/anie.200802376

[26] Heintzmann, R. and Ficz, G. (2013). Breaking the resolution limit in light microscopy. \textit{Methods Cell Biol.} 114 525-544.· doi:10.1016/B978-0-12-407761-4.00022-1

[27] Hell, S. W. (2007). Far-field optical nanoscopy. \textit{Science} 316 1153-1158.· doi:10.1126/science.1137395

[28] Hell, S. W. and Wichmann, J. (1994). Breaking the diffraction resolution limit by stimulated emission: Stimulated-emission-depletion fluorescence microscopy. \textit{Opt. Lett.} 19 780-782.· doi:10.22492/ijcs.1.1.03

[29] Helstrom, C. (1964). The detection and resolution of optical signals. \textit{IEEE Trans. Inf. Theory} 10 275-287. · Zbl 0126.35902

[30] Helstrom, C. (1965). Correction to 'The detection and resolution of optical signals'. \textit{IEEE Trans. Inf. Theory} 11 125. · Zbl 0126.35902 · doi:10.1109/TIT.1965.1053725

[31] Helstrom, C. W. (1973). Resolution of point sources of light as analyzed by quantum detection theory. \textit{IEEE Trans. Inf. Theory} 19 389-398.· doi:10.1109/TIT.1973.1055052

[32] Hess, S. T., Girirajan, T. P. K. and Mason, M. D. (2006). Ultra-high resolution imaging by fluorescence photoactivation localization microscopy. \textit{Biophys. J.} 91 4258-4272.· doi:10.1529/biophysj.106.091116

[33] Hohage, T. and Werner, F. (2016). Inverse problems with Poisson data: Statistical regularization theory, applications and algorithms. \textit{Inverse Probl.} 32 093001. · Zbl 1372.65163 · doi:10.1088/0266-5611/32/9/093001

[34] Houston, W. V. (1927). A compound interferometer for fine structure work. \textit{Phys. Rev.} 29 478-484.· doi:10.1103/PhysRev.29.478

[35] Huang, B., Bates, M. and Zhuang, X. (2009). Super-resolution fluorescence microscopy. \textit{Annu. Rev. Biochem.} 78 993-1016.· doi:10.1146/annurev.biochem.77.061906.092014

[36] Klar, T. A., Jakobs, S., Dyba, M., Egner, A. and Hell, S. W. (2000). Fluorescence microscopy with diffraction resolution barrier broken by stimulated emission. \textit{Proc. Natl. Acad. Sci. USA} 97 8206-8210.· doi:10.1073/pnas.97.15.8206

[37] Kulaitis, G., Munk, A. and Werner, F. (2021). Supplement to "What is resolution? A statistical minimax testing perspective on superresolution microscopy." https://doi.org/10.1214/20-AOS2037SUPP

[38] Lehmann, E. L. and Romano, J. P. (2005). \textit{Testing Statistical Hypotheses}, 3rd ed. \textit{Springer Texts in Statistics}. Springer, New York.

[39] Leonhardt, U. (2010). \textit{Essential Quantum Optics}: \textit{From Quantum Measurements to Black Holes}. Cambridge Univ. Press, Cambridge.· doi:10.1017/CBO9780511806117

[40] Lu, X.-M., Krovi, H., Nair, R., Guha, S. and Shapiro, J. H. (2018). Quantum-optimal detection of one-versus-two incoherent optical sources with arbitrary separation. \textit{npj Quantum Inf.} 4.· doi:10.1038/s41534-018-0114-y

[41] Middleton, D. (1953). Statistical criteria for the detection of pulsed carriers in noise. I. \textit{J. Appl. Phys.} 24 371-378. · Zbl 0052.36803

[42] Milanfar, P. and Shakouri, A. (2002). A statistical analysis of diffraction-limited imaging. In \textit{IEEE International Conference on Image Processing} 864-867.

[43] Minsky, M. (1961). Microscopy apparatus. US Patent 3013467, filed 7 November 1957, granted 19 December 1961.

[44] Morgenshtern, V. I. and Candès, E. J. (2016). Super-resolution of positive sources: The discrete setup. \textit{SIAM J. Imaging Sci.} 9 412-444. · Zbl 1352.49041 · doi:10.1137/15M1016552

[45] Munk, A., Staudt, T. and Werner, F. (2020). Statistical foundations of nanoscale photonic imaging. In \textit{Nanoscale Photonic Imaging} (T. Salditt, A. Egner and R. Luke, eds.) Springer, Berlin.

[46] Nair, R. and Tsang, M. (2016). Far-field superresolution of thermal electromagnetic sources at the quantum limit. \textit{Phys.}

Rev. Lett.} 117 190801.· doi:10.1103/PhysRevLett.117.190801

[47] Orfanidis, S. J. (2016). \textit{Electromagnetic Waves and Antennas}. Rutgers Univ. Available at http://eceweb1.rutgers.edu/texttildeloworfanidi/

[48] Oshikane, Y., Kataoka, T., Okuda, M., Hara, S., Inoue, H. and Nakano, M. (2007). Observation of nanostructure by scanning near-field optical microscope with small sphere probe. \textit{Sci. Technol. Adv. Mater.} 8 181-185.· doi:10.1016/j.stam.2007.02.013

[49] Pawley, J. B. (2006). \textit{Handbook of Biological Confocal Microscopy}, 3rd ed. Springer, New York.

[50] Ray, K. and Schmidt-Hieber, J. (2018). The Le Cam distance between density estimation, Poisson processes and Gaussian white noise. \textit{Math. Stat. Learn.} 1 101-170. · Zbl 1416.62054 · doi:10.4171/msl/1-2-1

[51] Reiffen, B. and Sherman, H. (1963). An optimum demodulator for Poisson processes: Photon source detectors. \textit{IEEE Trans. Inf. Theory} 51 1316-1320.

[52] Rittweger, E., Han, K. Y., Irvine, S. E., Eggeling, C. and Hell, S. W. (2009). STED microscopy reveals crystal colour centres with nanometric resolution. \textit{Nat. Photonics} 3 144-147.

[53] Rust, M. J., Bates, M. and Zhuang, X. (2006). Sub-diffraction-limit imaging by stochastic optical reconstruction microscopy (STORM). \textit{Nat. Methods} 3 793-795.· doi:10.1038/nmeth929

[54] Shahram, M. (2005). Statistical and information-theoretic analysis of resolution in imaging and array processing. Ph.D. thesis, Univ. California, Santa Cruz.

[55] Shahram, M. and Milanfar, P. (2004). Imaging below the diffraction limit: A statistical analysis. \textit{IEEE Trans. Image Process.} 13 677-689.· doi:10.1109/TIP.2004.826096

[56] Shahram, M. and Milanfar, P. (2006). Statistical and information-theoretic analysis of resolution in imaging. \textit{IEEE Trans. Inf. Theory} 52 3411-3437. · Zbl 1309.94031 · doi:10.1109/TIT.2006.878180

[57] Smith, S. T. (2005). Statistical resolution limits and the complexified Cramér-Rao bound. \textit{IEEE Trans. Signal Process.} 53 1597-1609. · Zbl 1370.94241 · doi:10.1109/TSP.2005.845426

[58] Staudt, T., Aspelmeier, T., Laitenberger, O., Geisler, C., Egner, A. and Munk, A. (2020). Statistical molecule counting in super-resolution fluorescence microscopy: Towards quantitative nanoscopy. \textit{Statist. Sci.} 35 92-111. · Zbl 07292498 · doi:10.1214/19-STS753

[59] Strutt, J. W. (1879). XXXI. Investigations in optics, with special reference to the spectroscope. \textit{Philos. Mag.} 8 261-274.· doi:10.1080/14786447908639684

[60] Tenne, R., Rossman, U., Rephael, B., Israel, Y., Krupinski-Ptaszek, A., Lapkiewicz, R., Silberberg, Y. and Oron, D. (2019). Super-resolution enhancement by quantum image scanning microscopy. \textit{Nat. Photonics} 13 116-122.· doi:10.1038/s41566-018-0324-z

[61] Terebizh, V. Yu. (1995). Image restoration with minimum a priori information. \textit{Usp. Fiz. Nauk} 165 143-176.· doi:10.3367/ufnr.0165.199502b.0143

[62] Tham, W.-K., Ferretti, H. and Steinberg, A. M. (2017). Beating Rayleigh's curse by imaging using phase information. \textit{Phys. Rev. Lett.} 118 070801.· doi:10.1103/PhysRevLett.118.070801

[63] Tsang, M., Nair, R. and Lu, X.-M. (2016). Quantum information for semiclassical optics. In \textit{Proceedings of SPIE} 10029, \textit{Quantum and Nonlinear Optics IV} 1002903.· doi:10.1117/12.2245733

[64] Tsang, M., Nair, R. and Lu, X. M. (2016). Quantum theory of superresolution for two incoherent optical point sources. \textit{Phys. Rev. X} 6 031033.· doi:10.1103/PhysRevX.6.031033

[65] von Diezmann, A., Shechtman, Y. and Moerner, W. E. (2017). Three-dimensional localization of single molecules for super-resolution imaging and single-particle tracking. \textit{Chem. Rev.} 117 7244-7275.· doi:10.1021/acs.chemrev.6b00629

[66] Westphal, V. and Hell, S. W. (2005). Nanoscale resolution in the focal plane of an optical microscope. \textit{Phys. Rev. Lett.} 94 143903.· doi:10.1103/PhysRevLett.94.143903

[67] Wouterlood, F. G. (2012). \textit{Cellular Imaging Techniques for Neuroscience and Beyond}. Academic Press, San Diego.

[68] Yamagata, K., Fujiwara, A. and Gill, R. D. (2013). Quantum local asymptotic normality based on a new quantum likelihood ratio. \textit{Ann. Statist.} 41 2197-2217 · Zbl 1278.81050 · doi:10.1214/13-AOS1147