Düster, Alexander; Hubrich, Simeon

Adaptive integration of cut finite elements and cells for nonlinear structural analysis. (English) Zbl 1481.74695

De Lorenzis, Laura (ed.) et al., Modeling in engineering using innovative numerical methods for solids and fluids. Papers based on the presentations at the CISM course, Udine, Italy, October 15–19, 2018. Cham: Springer. CISM Courses Lect. 599, 31-73 (2020).

Summary: Fictitious domain methods facilitate the discretization of boundary value problems by applying simple meshes containing finite elements or cells that do not conform to the geometry of the domain of interest. In this way, the effort of meshing complex domains is shifted to the numerical integration of those elements/cells that are cut by the boundary of the domain. In this chapter, we will first introduce a high-order fictitious domain method and then present adaptive methods that are suited for the numerical integration of broken elements and cells. Since the quadrature schemes presented in this chapter are quite general, they can be applied to the different versions of fictitious domain methods.

For the entire collection see [Zbl 1470.76007].

MSC:

74S05 Finite element methods applied to problems in solid mechanics
74C05 Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)
74C15 Large-strain, rate-independent theories of plasticity (including nonlinear plasticity)
74F10 Fluid-solid interactions (including aero- and hydro-elasticity, porosity, etc.)
74K20 Plates

Keywords:

finite cell method; finite element method; fictitious domain method; quadrature scheme; small-strain elastoplasticity; finite-strain elastoplasticity; plate

Full Text: DOI

References:

[1] Abedian, A., \& Düster, A. (2019). Equivalent Legendre polynomials: Numerical integration of discontinuous functions in the finite element methods. Computer Methods in Applied Mechanics and Engineering, 343, 690-720. https://doi.org/10.1016/j.cma.2018.08.002. · Zbl 1440.65166 · doi:10.1016/j.cma.2018.08.002

[2] Abedian, A., Parvizian, J., Düster, A., Khademyzadeh, H., \& Rank, E. (2013). Performance of different integration schemes in facing discontinuities in the finite cell method. International Journal of Computational Methods, 10(3), 1350002/1-24. https://doi.org/10.1142/S0219876213500023. · Zbl 1359.65245 · doi:10.1142/S0219876213500023

[3] Abedian, A., Parvizian, J., Düster, A., \& Rank, E. (2013). The finite cell method for the \(J_2\) flow theory of plasticity. Finite Elements in Analysis and Design, 69, 37-47. · Zbl 1359.65245 · doi:10.1016/j.finel.2013.01.006

[4] Abedian, A., Parvizian, J., Düster, A., \& Rank, E. (2014). Finite cell method compared to \(h\)-version finite element method for elasto-plastic problems. Applied Mathematics and Mechanics, 35(10), 1239-1248. https://doi.org/10.1007/s10483-014-1861-9.· doi:10.1007/s10483-014-1861-9

[5] Burman, E., \& Hansbo, P. (2010). Fictitious domain finite element methods using cut elements: I. A stabilized Lagrange multiplier method. Computer Methods in Applied Mechanics and Engineering, 199(41-44), 2680-2686. · Zbl 1231.65207 · doi:10.1016/j.cma.2010.05.011

[6] Burman, E., \& Hansbo, P. (2012). Fictitious domain finite element methods using cut elements: II. A stabilized Nitsche method. Applied Numerical Mathematics, 62(4), 328-341. https://doi.org/10.1016/j.apnum.2011.01.008. · Zbl 1316.65099 · doi:10.1016/j.apnum.2011.01.008

[7] Burman, E., Claus, S., Hansbo, P., Larson, M. G., \& Massing, A. (2015). CutFEM: Discretizing geometry and partial differential equations. International Journal for Numerical Methods in Engineering, 104, 472-501. · Zbl 1352.65604 · doi:10.1002/nme.4823

[8] Cottrell, J. A., Hughes, T. J. R., \& Bazilevs, Y. (2009). Isogeometric analysis: Towards integration of CAD and FEM.

Hoboken: Wiley. ISBN 978-0-470-74873-2. · Zbl 1378.65009

[9] Dauge, M., Düster, A., \& Rank, E. (2015). Theoretical and numerical investigation of the finite cell method. Journal of Scientific Computing, 65, 1039-1064. https://doi.org/10.1007/s10915-015-9997-3. · Zbl 1331.65160 · doi:10.1007/s10915-015-9997-3

[10] de Souza Neto, E. A., Perić, D., \& Owen, D. R. J. (2008). Computational methods for plasticity, theory and applications. Hoboken: Wiley. ISBN 978-0-470-69452-7.

[11] Del Pino, S., \& Pironneau, O. (2003). A fictitious domain based general pde solver. In P. Neittanmaki, Y. Kuznetsov \& O. Pironneau (Eds.), Numerical methods for scientific computing variational problems and applications, CIMNE, Barcelona, Spain.

[12] Düster, A., \& Allix, O. (2019). Selective enrichment of moment fitting and application to cut finite elements and cells. Computational Mechanics. https://doi.org/10.1007/s00466-019-01776-2.· doi:10.1007/s00466-019-01776-2

[13] Düster, A., \& Rank, E. (2002). A p-version finite element approach for two- and three-dimensional problems of the \({J}_2\) flow theory with non-linear isotropic hardening. International Journal for Numerical Methods in Engineering, 53, 49-63. · Zbl 1112.74509 · doi:10.1002/nme.391

[14] Düster, A., Niggl, A., Nübel, V., \& Rank, E. (2002). A numerical investigation of high-order finite elements for problems of elasto-plasticity. Journal of Scientific Computing, 17, 429-437. · Zbl 1011.74064 · doi:10.1023/A:1015189706770

[15] Düster, A., Parvizian, J., Yang, Z., \& Rank, E. (2008). The finite cell method for three-dimensional problems of solid mechanics. Computer Methods in Applied Mechanics and Engineering, 197, 3768-3782. · Zbl 1194.74517 · doi:10.1016/j.cma.2008.02.036

[16] Düster, A., Sehlhorst, H.-G., \& Rank, E. (2012). Numerical homogenization of heterogeneous and cellular materials utilizing the finite cell method. Computational Mechanics, 50, 413-431. https://doi.org/10.1007/s00466-012-0681-2. · Zbl 1386.74117 · doi:10.1007/s00466-012-0681-2

[17] Düster, A., Rank, E., \& Szabó, B. (2017). The p-Version of the Finite Element and Finite Cell Methods. In E. Stein, R. de Borst, \& T. J. R. Hughes (Eds.), Encyclopedia of computational mechanics, 2nd edn, vol Part 1. Solids and Structures (Chap. 4, pp. 137-171). Hoboken: Wiley. https://doi.org/10.1002/9781119176817.ecm2003g. ISBN 978-1-119-00379-3.

[18] Fries, T.-P., \& Omerović, S. (2016). Higher-order accurate integration of implicit geometries. International Journal for Numerical Methods in Engineering, 106(5), 323-371. · Zbl 1352.65498 · doi:10.1002/nme.5121

[19] Glowinski, R., \& Kuznetsov, Y. (2007). Distributed Lagrange multipliers based on fictitious domain method for second order elliptic problems. Computer Methods in Applied Mechanics and Engineering, 196, 1498-1506. · Zbl 1173.65369 · doi:10.1016/j.cma.2006.05.013

[20] Gnegel, S. (2019). The finite cell method for the computation of cellular materials. Ph.D. thesis, Fachgebiet für Numerische Strukturanalyse mit Anwendungen in der Schiffstechnik (M-10), TU Hamburg.

[21] Heinze, S., Joulaian, M., Egger, H., \& Düster, A. (2014). Efficient computation of cellular materials using the finite cell method. Proceedings in Applied Mathematics and Mechanics, 14, 251-252. https://doi.org/10.1002/pamm.201410113.· doi:10.1002/pamm.201410113

[22] Hubrich, S., \& Düster, A. (2018). Adaptive numerical integration of broken finite cells based on moment fitting applied to finite strain problems. Proceedings in Applied Mathematics and Mechanics, 18, e201800089. https://doi.org/10.1002/pamm.201800089. · Zbl 1442.65024

[23] Hubrich, S., \& Düster, A. (2019). Numerical integration for nonlinear problems of the finite cell method using an adaptive scheme based on moment fitting. Computers \& Mathematics with Applications, 77, 1983-1997. https://doi.org/10.1016/j.camwa.2018.11.030. · Zbl 1442.65024 · doi:10.1016/j.camwa.2018.11.030

[24] Hubrich, S., Di Stolfo, P., Kudela, L., Kollmannsberger, S., Rank, E., Schröder, A., et al. (2017). Numerical integration of discontinuous functions: Moment fitting and smart octree. Computational Mechanics, 60, 863-881. https://doi.org/10.1007/s00466-017-1441-0. · Zbl 1387.65025 · doi:10.1007/s00466-017-1441-0

[25] Hughes, T. J. R., Cottrell, J. A., \& Bazilevs, Y. (2005). Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. Computer Methods in Applied Mechanics and Engineering, 194, 4135-4195. · Zbl 1151.74419 · doi:10.1016/j.cma.2004.10.008

[26] Jomo, J. N., Zander, N., Elhaddad, M., Özcan, A., Kollmannsberger, S., Mundani, R.-P., et al. (2017). Parallelization of the multi-level hp-adaptive finite cell method. Computers \& Mathematics with Applications, 74, 126-142. https://doi.org/10.1016/j.camwa.2017.01.004 · Zbl 1375.65151 · doi:10.1016/j.camwa.2017.01.004

[27] Joulaian, M., \& Düster, A. (2013). Local enrichment of the finite cell method for problems with material interfaces. Computational Mechanics, 52, 741-762. https://doi.org/10.1007/s00466-013-0853-8. · Zbl 1311.74123 · doi:10.1007/s00466-013-0853-8

[28] Joulaian, M., Hubrich, S., \& Düster, A. (2016). Numerical integration of discontinuities on arbitrary domains based on moment fitting. Computational Mechanics, 57, 979-999. https://doi.org/10.1007/s00466-016-1273-3. · Zbl 1382.65066 · doi:10.1007/s00466-016-1273-3

[29] Kollmannsberger, S., Özcan, A., Baiges, J., Ruess, M., Rank, E., \& Reali, A. (2014). Parameter-free, weak imposition of Dirichlet boundary conditions and coupling of trimmed and non-conforming patches. International Journal for Numerical Methods in Engineering, 101(9), 1-30. https://doi.org/10.1002/nme.4817. · Zbl 1352.65520 · doi:10.1002/nme.4817

[30] Kudela, L., Zander, N., Bog, T., Kollmannsberger, S., \& Rank, E. (2015). Efficient and accurate numerical quadrature for immersed boundary methods. Advanced Modeling and Simulation in Engineering Sciences, 2(1), 1-22. https://doi.org/10.1186/s40323-015-0031-y. ISSN 2213-7467.

[31] Loehnert, S., Mueller-Hoeppe, D. S., \& Wriggers, P. (2011). 3D corrected XFEM approach and extension to finite deformation theory. International Journal for Numerical Methods in Engineering, 86, 431-452. · Zbl 1216.74026 · doi:10.1002/nme.3045

[32] Lyness, J. N., \& Jespersen, D. (1975). Moderate degree symmetric quadrature rules for the triangle. Journal of the Institute

of Mathematics and Its Applications, 15, 19-32. · Zbl 0297.65018 · doi:10.1093/imamat/15.1.19

[33] Lyness, J. N., \& Monegato, G. (1977). Quadrature rules for regions having regular hexagonal symmetry. SIAM Journal on Numerical Analysis, 14, 283-295. · Zbl 0365.65014 · doi:10.1137/0714018

[34] Melenk, J. M., \& Babuška, I. (1996). The partition of unity finite element method: Basic theory and applications. Computer Methods in Applied Mechanics and Engineering, 139, 289-314. · Zbl 0881.65099 · doi:10.1016/S0045-7825(96)01087-0

[35] Mittal, R., \& Iaccarino, G. (2005). Immersed boundary method. Annual Review Fluid Mechanics, 37, 239-260. · Zbl 1117.76049 · doi:10.1146/annurev.fluid.37.061903.175743

[36] Mousavi, S. E., \& Sukumar, N. (2010). Generalized Gaussian quadrature rules for discontinuities and crack singularities in the extended finite element method. Computer Methods in Applied Mechanics and Engineering, 199(49-52), 3237-3249. https://doi.org/10.1016/j.cma.2010.06.031. · Zbl 1225.74099 · doi:10.1016/j.cma.2010.06.031

[37] Mousavi, S. E., \& Sukumar, N. (2011). Numerical integration of polynomials and discontinuous functions on irregular convex polygons and polyhedrons. Computational Mechanics, 47, 535-554. · Zbl 1221.65078 · doi:10.1007/s00466-010-0562-5

[38] Müller, B., Kummer, F., \& Oberlack, M. (2013). Highly accurate surface and volume integration on implicit domains by means of moment-fitting. International Journal for Numerical Methods in Engineering, 96, 512-528. https://doi.org/10.1002/nme.4569. · Zbl 1352.65083 · doi:10.1002/nme.4569

[39] Neittaanmäki, P., \& Tiba, D. (1995). An embedding of domains approach in free boundary problems and optimal design. SIAM Journal on Control and Optimization, 33(5), 1587-1602. · Zbl 0843.49024 · doi:10.1137/S0363012992231124

[40] Parvizian, J., Düster, A., \& Rank, E. (2007). Finite cell method - h- and p-extension for embedded domain problems in solid mechanics. Computational Mechanics, 41, 121-133. · Zbl 1162.74506 · doi:10.1007/s00466-007-0173-y

[41] Peskin, C. (2002). The immersed boundary method. Acta Numerica, 11, 1-39. · Zbl 1123.74309 · doi:10.1017/S0962492902000077

[42] Press, W. H., Teukolsky, S. A., Vetterling, W. T., \& Flannery, B. P. (2002). Numerical recipes in C++. The art of scientific computing(2nd ed.). Cambridge: Cambridge University Press. ISBN 0-521-75033-4. · Zbl 1078.65500

[43] Ramière, I., Angot, P., \& Belliard, M. (2007). A fictitious domain approach with spread interface for elliptic problems with general boundary conditions. Computer Methods in Applied Mechanics and Engineering, 196, 766-781. · Zbl 1121.65364 · doi:10.1016/j.cma.2006.05.012

[44] Ruess, M., Schillinger, D., Bazilevs, Y., Varduhn, V., \& Rank, E. (2013). Weakly enforced essential boundary conditions for NURBS-embedded and trimmed NURBS geometries on the basis of the finite cell method. International Journal for Numerical Methods in Engineering, 95(10), 811-846. https://doi.org/10.1002/nme.4522. · Zbl 1352.65643 · doi:10.1002/nme.4522

[45] Samet, H. (1990). Applications of spatial data structures: Computer graphics, image processing, and GIS. Boston, MA: Addison-Wesley Longman Publishing Co., Inc.

[46] Saul'ev, V. K. (1963a). A method for automatization of the solution of boundary value problems on high performance computers. Doklady Akademii Nauk SSSR, 144(1962), 497-500 (in Russian). English translation in Soviet Mathematics Doklady, 3, 763-766.

[47] Saul'ev, V. K. (1963). On solution of some boundary value problems on high performance computers by fictitious domain method. Siberian Mathematical Journal, 4, 912-925.

[48] Schillinger, D., \& Ruess, M. (2015). The finite cell method: A review in the context of higher-order structural analysis of CAD and image-based geometric models. Archives of Computational Methods in Engineering, 22, 391-455. https://doi.org/10.1007/s11831-014-9115-y. · Zbl 1348.65056 · doi:10.1007/s11831-014-9115-y

[49] Schillinger, D., Ruess, M., Zander, N., Bazilevs, Y., Düster, A., \& Rank, E. (2012). Small and large deformation analysis with the p- and B-spline versions of the finite cell method. Computational Mechanics, 50, 445-478. https://doi.org/10.1007/s00466-012-0684-z. · Zbl 1398.74401 · doi:10.1007/s00466-012-0684-z

[50] Schwarz, H. R. (2004). Numerische Mathematik(5th ed). B.G. Teubner. ISBN 978-3519429609.

[51] Simo, J. C., \& Hughes, T. J. R. (1998). Computational inelasticity. Berlin: Springer. · Zbl 0934.74003

[52] Stein, E. (Ed.). (2002). Error-controlled adaptive finite elements in solid mechanics. Hoboken: Wiley.

[53] Strouboulis, T., Copps, K., \& Babuška, I. (2000). The generalized finite element method: An example of its implementation and illustration of its performance. International Journal for Numerical Methods in Engineering, 47, 1401-1417. · Zbl 0955.65080 · doi:10.1002/(SICI)1097-0207(20000320)47:8<1401::AID-NME835>3.0.CO;2-8

[54] Strouboulis, T., Copps, K., \& Babuška, I. (2001). The generalized finite element method. Computer Methods in Applied Mechanics and Engineering, 190, 4081-4193. · Zbl 0997.74069 · doi:10.1016/S0045-7825(01)00188-8

[55] Szabó, B. A., \& Babuška, I. (1991). Finite element analysis. Hoboken: Wiley. ISBN 0-471-50273-1. · Zbl 0792.73003

[56] Szabó, B. A., Düster, A., \& Rank, E. (2004). The p-version of the finite element method. In E. Stein, R. de Borst, \& T. J. R. Hughes (Eds.), Encyclopedia of computational mechanics(Vol 1, Chap. 5, pp. 119-139). Hoboken: Wiley. https://doi.org/10.1002/0470091355.ecm003g. ISBN 0-470-84699-2.

[57] Taghipour, A., Parvizian, J., Heinze, S., \& Düster, A. (2018). The finite cell method for nearly incompressible finite strain plasticity problems with complex geometries. Computers \& Mathematics with Applications, 75, 3298-3316. https://doi.org/10.1016/j.camwa.2018.01.0 · Zbl 1409.74047 · doi:10.1016/j.camwa.2018.01.048

[58] Ventura, G. (2006). On the elimination of quadrature subcells for discontinuous functions in the eXtended finite-element method. International Journal for Numerical Methods in Engineering, 66, 761-795. · Zbl 1110.74858 · doi:10.1002/nme.1570

[59] Ventura, G., \& Benvenuti, E. (2015). Equivalent polynomials for quadrature in Heaviside function enrichment elements. International Journal for Numerical Methods in Engineering, 102, 688-710. · Zbl 1352.65559 · doi:10.1002/nme.4679

[60] Wriggers, P. (2008). Nonlinear finite-element-methods. Berlin: Springer. ISBN 3-540-71000-0. · Zbl 1153.74001

[61] Zander, N., Bog, T., Elhaddad, M., Frischmann, F., Kollmannsberger, S., \& Rank, E. (2016). The multi-level hp-method for three-dimensional problems: Dynamically changing high-order mesh refinement with arbitrary hanging nodes. Computer Methods in Applied Mechanics and Engineering, 310, 252-277. https://doi.org/10.1016/j.cma.2016.07.007 · Zbl 1439.65201 · doi:10.1016/j.cma.2016.07.007