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One- and two-hump solutions of a singularly perturbed cubic nonlinear Schrödinger equation. (English) Zbl 1478.35190

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Summary: The paper considers the existence of one- or two-hump solutions of a singularly perturbed nonlinear Schrödinger (NLS) equation, which is the standard NLS equation with a third order perturbation. In particular, this equation appears in the field of nonlinear optics, where it is used to describe pulses in optical fibers near a zero dispersion wavelength. It has been shown formally and numerically that the perturbed NLS equation has one- or multi-hump solutions with small oscillations at infinity, called generalized one- or multi-hump solutions. The main purpose of the paper is to provide the first rigorous proof of the existence of generalized one- or two-hump solutions of the singularly perturbed NLS equation. The several invariant properties of the equation, i.e., the translational invariance, the gauge invariance and the reversibility property, are essential to obtain enough free constants to prove the existence. The ideas and methods presented here may be applicable to show existence of generalized 2^k-hump solutions of the equation.

MSC:

35Q55 NLS equations (nonlinear Schrödinger equations)
78A60 Lasers, masers, optical bistability, nonlinear optics
35B25 Singular perturbations in context of PDEs
35A01 Existence problems for PDEs: global existence, local existence, non-existence

Full Text: DOI

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