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High-accuracy time discretization of stochastic fractional diffusion equation. (English)

Zbl 07435363
J. Sci. Comput. 90, No. 1, Paper No. 19, 24 p. (2022)

Summary: A high-accuracy time discretization is discussed to numerically solve the nonlinear fractional diffusion equation forced by a space-time white noise. The main purpose of this paper is to improve the temporal convergence rate by modifying the semi-implicit Euler scheme. The solution of the equation is only Hölder continuous in time, which is disadvantageous to improve the temporal convergence rate. Firstly, the system is transformed into an equivalent form having better regularity than the original one in time. But the regularity of nonlinear term remains unchanged. Then, combining Lagrange mean value theorem and independent increments of Brownian motion leads to a higher accuracy discretization of nonlinear term which ensures the implementation of the proposed time discretization scheme without loss of convergence rate. Our scheme can improve the convergence rate from $\min\{\frac{\gamma}{2\alpha},\frac{1}{2}\}$ to $\min\{\frac{\gamma}{\alpha},1\}$ in the sense of mean-squared L^2-norm. The theoretical error estimates are confirmed by extensive numerical experiments.

MSC:

65Mxx Numerical methods for partial differential equations, initial value and time-dependent initial-boundary value problems
26A33 Fractional derivatives and integrals
65M60 Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs
65L20 Stability and convergence of numerical methods for ordinary differential equations
65C30 Numerical solutions to stochastic differential and integral equations

Keywords:

high-accuracy time discretization; modifying the semi-implicit Euler scheme; the regularity of nonlinear term; mean-squared L^2-norm

Full Text: DOI arXiv

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