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Unconditional stability and optimal error estimates of Euler implicit/explicit-SAV scheme for the Navier-Stokes equations. (English) Zbl 07435294

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Summary: The unconditional stability and convergence analysis of the Euler implicit/explicit scheme with finite element discretization are studied for the incompressible time-dependent Navier-Stokes equations based on the scalar auxiliary variable approach. Firstly, a corresponding equivalent system of the Navier-Stokes equations with three variables is formulated, the stable finite element spaces are adopted to approximate these variables and the corresponding theoretical analysis results are provided. Secondly, a fully discrete scheme based on the backward Euler method is developed, the temporal treatment is based on the Euler implicit/explicit scheme, which is implicit for the linear terms and explicit for the nonlinear term. Hence, a constant coefficient algebraic system is formed and it can be solved efficiently. The discrete unconditional energy dissipation and stability of numerical solutions in various norms are established with any restriction on the time step, optimal error estimates are also provided. Finally, some numerical results are provided to illustrate the performances of the considered numerical scheme.

MSC:

65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs

Keywords:

time-dependent Navier-Stokes equations; Euler explicit/implicit scheme; scalar auxiliary variable; unconditional stability; optimal error estimates

Software:

XTOR; XTOR-2F

Full Text: DOI

References:

[1] Ammi, AA; Marion, M., Nonlinear Galerkin methods and mixed finite elements: two-grid algorithms for the Navier-Stokes equations, Numer. Math., 68, 189-213 (1994) · Zbl 0811.76035 · doi:10.1007/s002110050056

[2] Dubois, T.; Jauberteau, F.; Temam, R., Solution of the incompressible Navier-Stokes equations by the nonlinear Galerkin method, J. Sci. Comput., 8, 167-194 (1993) · Zbl 0783.76068 · doi:10.1007/BF01060871

[3] Erturk, E., Corke, T.C., GÄokcÄol, C.: Numerical solutions of 2-D steady incompressible driven cavity flow at high Reynolds numbers. Int. J. Numeri. Methods Fluids 48, 747-774 (2005) · Zbl 1071.76038

[4] Ghia, U.; Ghia, KN; Shin, CT, High-Re solutions for incompressibel flow using the Navier-Stokes equations and a multigrid method, J. Comput. Phys., 48, 387-411 (1982) · Zbl 0511.76031 · doi:10.1016/0021-9991(82)90058-4

[5] Giraldo, FX; Restelli, M.; Laeuter, M., Semi-implicit formulations of the Navier-Stokes equations: application to nonhydrostatic atmospheric modeling, SIAM J. Sci. Comput., 32, 3394-3425 (2010) · Zbl 1237.76153 · doi:10.1137/090775889

[6] Girault, V.; Raviart, PA, Finite Element Methods for the the Navier-Stokes Equations (1986), Berlin: Spinger-Verlag, Berlin · Zbl 0585.65077 · doi:10.1007/978-3-642-61623-5

[7] Hansen, E.; Stillfjord, T., Convergence of the implicit-explicit Euler scheme applied to perturbed dissipative evolution equations, Math. Comput., 82, 284, 1975-1985 (2013) · Zbl 1282.65059 · doi:10.1090/S0025-5718-2013-02702-0

[8] He, YN, A fully discrete stabilized finite-element method for the time-dependent Navier-Stokes problem, IMA J. Numer. Anal., 23, 665-691 (2003) · Zbl 1135.76331 · doi:10.1093/imanum/23.4.665

[9] He, YN, The Euler implicit/explicit scheme for the 2D time-dependent Navier-Stokes equations with smooth or non-smooth initial data, Math. Comput., 77, 264, 2097-2124 (2008) · Zbl 1198.65222 · doi:10.1090/S0025-5718-08-02127-3

[10] He, YN, Unconditional convergence of the Euler semi-implicit scheme for the three-dimensional incompressible MHD equations, IMA J. Numer. Anal., 35, 767-801 (2015) · Zbl 1312.76061 · doi:10.1093/imanum/dru015

[11] He, YN; Huang, PZ; Feng, XL, \(H^2\)-stability of first order fully discrete schemes for the time-dependent Navier-Stokes equations, J. Sci. Comput., 62, 230-264 (2015) · Zbl 1334.76078 · doi:10.1007/s10915-014-9854-9

[12] He, YN; Li, KT, Two-level stabilized finite element methods for the steady Navier-Stokes problem, Computing, 74, 337-351 (2005) · Zbl 1099.65111 · doi:10.1007/s00607-004-0118-7

[13] He, YN; Li, J., A penalty finite element method based on the Euler implicit/explicit scheme for the time-dependent Navier-Stokes equations, J. Comput. Appl. Math., 235, 3, 708-725 (2010) · Zbl 1277.76037 · doi:10.1016/j.cam.2010.06.025

[14] Heywood, J.; Rannacher, R., Finite element approximation of the nonstantionary Navier-Stokes problem I: Regularity of solutions and second-order error estimates for spatial discretization, SIAM J. Numer. Anal., 19, 275-311 (1982) · Zbl 0487.76035 · doi:10.1137/0719018

[15] Hill, AT; Süli, E., Approximation of the global attractor for the incompressible Navier-Stokes equations, IMA J. Numer. Anal., 20, 633-667 (2000) · Zbl 0982.76022 · doi:10.1093/imanum/20.4.633

[16] Hou, T.; Shi, ZQ, An efficient semi-implicit immersed boundary method for the Navier-Stokes equations, J. Comput. Phys., 227, 8968-8991 (2008) · Zbl 1161.76048 · doi:10.1016/j.jcp.2008.07.005

[17] Kaya, S.; Rivière, B., A two-grid stabilization method for solving the steady-state Navier-Stokes equations, Numer. Methods Partial Differ. Equ., 22, 728-743 (2006) · Zbl 1089.76034 · doi:10.1002/num.20120

[18] Li, BY; Sun, WW, Error analysis of linearized semi-implicit Galerkin finite element methods for nonlinear parabolic equations, Int. J. Numer. Anal. Model., 10, 3, 622-633 (2013) · Zbl 1281.65122

[19] Li, XL; Shen, J., Error analysis of the SAV-MAC scheme for the Navier-Stokes equations, SIAM J. Numer. Anal., 58, 5, 2465-2491 (2020) · Zbl 1452.65167 · doi:10.1137/19M1288267

[20] Li, XL; Shen, J., On a SAV-MAC scheme for the Cahn-Hilliard-Navier-Stokes phase-field model and its error analysis for the corresponding Cahn-Hilliard-Stokes case, Math. Models Methods Appl. Sci., 30, 12, 2263-2297 (2020) · Zbl 1471.65106 · doi:10.1142/S0218202520500438

[21] Li, XL; Shen, J.; Rui, HX, Energy stability and convergence of SAV block-centered finite difference methd for gradient flows, Math. Comput., 88, 319, 2047-2068 (2019) · Zbl 1422.65165 · doi:10.1090/mcom/3428

[22] Lin, LL; Yang, ZG; Dong, SC, Numerical approximation of incompressible Navier-Stokes equations based on an auxiliary energy variable, J. Comput. Phys., 388, 1-22 (2019) · Zbl 1452.76093 · doi:10.1016/j.jcp.2019.03.012

[23] Luetjens, H.; Luciani, JF, XTOR-2F: A fully implicit Newton-Krylov solver applied to nonlinear 3D extended MHD in tokamaks, J. Comput. Phys., 229, 8130-8143 (2010) · Zbl 1220.76055 · doi:10.1016/j.jcp.2010.07.013

[24] Marti, J.; Ryzhakov, PB, An explicit-implicit finite element model for the numerical solution of incompressible Navier-Stokes equations on moving grids, Comput. Methods Appl. Mech. Eng., 350, 750-765 (2019) · Zbl 1441.76065 · doi:10.1016/j.cma.2019.03.007

[25] Nguyen, NC; Peraire, J.; Cockburn, B., An implicit high-order hybridizable discontinuous Galerkin method for nonlinear convection-diffusion equations, J. Comput. Phys., 228, 8841-8855 (2009) · Zbl 1177.65150 · doi:10.1016/j.jcp.2009.08.030

[26] Shen, J., Long time stability and convergence for fully discrete nonlinear Galerkin methods, Appl. Anal., 38, 201-229 (1990) · Zbl 0684.65095 · doi:10.1080/00036819008839963

[27] Shen, J., On error estimates of projection methods for Navier-Stokes equations: first-order schemes, SIAM J. Numer. Anal., 29, 57-77 (1992) · Zbl 0741.76051 · doi:10.1137/0729004

[28] Shen, J., On error estimates of some higher order projection and penalty-projection methods for Navier-Stokes equations, Numer. Math., 62, 49-73 (1992) · Zbl 0782.76025 · doi:10.1007/BF01396220

[29] Shen, J., On error estimates of the projection, methods for the Navier-Stokes equations: second-order schemes, Math. Comput., 65, 1039-1066 (1996) · Zbl 0855.76049 · doi:10.1090/S0025-5718-96-00750-8

[30] Shen, J.; Xu, J., Convergence and error analysis for the scalar auxiliary variable (SAV) schemes to gradient flows, SIAM J. Numer. Anal., 56, 2895-2912 (2018) · Zbl 1403.65047 · doi:10.1137/17M1159968

[31] Shen, J.; Xu, J.; Yang, J., The scalar auxiliary variable (SAV) approach for gradient flows, J. Comput. Phys., 353, 407-416 (2018) · Zbl 1380.65181 · doi:10.1016/j.jcp.2017.10.021

[32] Shi, H.; Li, Y., Local discontinuous Galerkin methods with implicit-explicit multistep time-marching for solving the nonlinear Cahn-Hilliard equation, J. Comput. Phys., 394, 719-731 (2019) · Zbl 1452.65249 · doi:10.1016/j.jcp.2019.05.040

[33] Su, J.; He, YN, The almost unconditional convergence of the Euler implicit/explicit scheme for the three dimensional non-stationary Navier-Stokes equations, Discrete Contin. Dyn. Syst.-B, 22, 9, 3421-3438 (2017) · Zbl 1368.35158

[34] Temam, R., Navier-Stokes Equations, Theory and Numerical Analysis (1984), Amsterdam, New York, Oxford: North-Holland, Amsterdam, New York, Oxford · Zbl 0568.35002

[35] Tone, F.; Wirosoetisno, D., On the long-time stability of the implicit Euler scheme for the two-dimensional Navier-Stokes equations, SIAM J. Numer. Anal., 44, 29-40 (2006) · Zbl 1108.76050 · doi:10.1137/040618527

[36] Yang, XF; Zhao, J.; Wang, Q., Numerical approximations for the molecular beam epitaxial growth model based on the invariant energy quadratization method, J. Comput. Phys., 333, 104-127 (2017) · Zbl 1375.82121 · doi:10.1016/j.jcp.2016.12.025

[37] Yang, XF; Zhao, J.; He, XM, Linear, second order and unconditionally energy stable schemes for the viscous Cahn-Hilliard equation with hyperbolic relaxation using the invariant energy quadratization method, J. Comput. Appl. Math., 343, 80-97 (2018) · Zbl 1462.65117 · doi:10.1016/j.cam.2018.04.027

[38] Yang, ZG; Dong, SC, An unconditionally energy-stable scheme based on an implicit auxiliary energy variable for incompressible two-phase flows with different densities involving only precomputable coefficient matrices, J. Comput. Phys., 393, 229-257 (2019) · Zbl 1452.76244 · doi:10.1016/j.jcp.2019.05.018

[39] Zhao, J.; Wang, Q.; Yang, XF, Numerical approximations for a phase field dendritic crystal growth model based on the invariant energy quadratization approach, Int. J. Numer. Meth. Eng., 110, 3, 279-300 (2017) · Zbl 1365.74138 · doi:10.1002/nme.5372

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