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Various soliton solutions and asymptotic state analysis for the discrete modified Korteweg-de Vries equation. (English) [Zbl 1479.35741](#)

Adv. Math. Phys. 2021, Article ID 3445894, 22 p. (2021).

Summary: Under investigation is the discrete modified Korteweg-de Vries (mKdV) equation, which is an integrable discretization of the continuous mKdV equation that can describe some physical phenomena such as dynamics of anharmonic lattices, solitary waves in dusty plasmas, and fluctuations in nonlinear optics. Through constructing the discrete generalized $(m, N - m)$ -fold Darboux transformation for this discrete system, the various discrete soliton solutions such as the usual soliton, rational soliton, and their mixed soliton solutions are derived. The elastic interaction phenomena and physical characteristics are discussed and illustrated graphically. The limit states of diverse soliton solutions are analyzed via the asymptotic analysis technique. Numerical simulations are used to display the dynamical behaviors of some soliton solutions. The results given in this paper might be helpful for better understanding the physical phenomena in plasma and nonlinear optics.

MSC:

[35Q53](#) KdV equations (Korteweg-de Vries equations)

[35Q51](#) Soliton equations

[35C08](#) Soliton solutions

[35B40](#) Asymptotic behavior of solutions to PDEs

[37K10](#) Completely integrable infinite-dimensional Hamiltonian and Lagrangian systems, integration methods, integrability tests, integrable hierarchies (KdV, KP, Toda, etc.)

[37K35](#) Lie-Bäcklund and other transformations for infinite-dimensional Hamiltonian and Lagrangian systems

[82D10](#) Statistical mechanics of plasmas

[78A60](#) Lasers, masers, optical bistability, nonlinear optics

Keywords:

[Korteweg-de Vries equation; solitary waves in dusty plasmas; fluctuations in nonlinear optics](#)

Software:

[Matlab](#)

Full Text: [DOI](#)

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