**Goodrich, Christopher S.**

**A one-dimensional Kirchhoff equation with generalized convolution coefficients.** (English)

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Summary: For $q \geq 1$ we consider the one-dimensional Kirchhoff-type problem

$$-A((a * (u')^q)(1))u''(t) = \lambda f(t, u(t)), \quad t \in (0, 1),$$

where $a * (u')^q$ represents a finite convolution, subject to right-focal boundary conditions. Because the nonlocal coefficient is phrased in terms of convolution the results of this paper can accommodate all manner of nonlocal coefficients, such as a fractional derivative coefficient of Caputo type. A nonstandard order cone together with a specially tailored open set is used to deduce existence of at least one positive solution for this problem via topological fixed point theory.

**MSC:**

33B15 Gamma, beta and polygamma functions
34B10 Nonlocal and multipoint boundary value problems for ordinary differential equations
34B18 Positive solutions to nonlinear boundary value problems for ordinary differential equations
42A85 Convolution, factorization for one variable harmonic analysis
44A35 Convolution as an integral transform
26D15 Inequalities for sums, series and integrals
35J25 Boundary value problems for second-order elliptic equations
35J60 Nonlinear elliptic equations
47H07 Monotone and positive operators on ordered Banach spaces or other ordered topological vector spaces
47H30 Particular nonlinear operators (superposition, Hammerstein, Nemytskiĭ, Uryson, etc.)

**Keywords:**

finite convolution; Kirchhoff equation; Caputo fractional derivative; positive solution; coercivity

**Full Text:** DOI

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