Lasarzik, Robert

Maximally dissipative solutions for incompressible fluid dynamics. (English) Zbl 07432866

Z. Angew. Math. Phys. 73, No. 1, Paper No. 1, 21 p. (2022)

Summary: We introduce the new concept of maximally dissipative solutions for a general class of isothermal GENERIC systems. Under certain assumptions, we show that maximally dissipative solutions are well-posed as long as the bigger class of dissipative solutions is non-empty. Applying this result to the Navier-Stokes and Euler equations, we infer global well-posedness of maximally dissipative solutions for these systems. The concept of maximally dissipative solutions coincides with the concept of weak solutions as long as the weak solutions inherits enough regularity to be unique.

MSC:

35Qxx Partial differential equations of mathematical physics and other areas of application
35D99 Generalized solutions to partial differential equations
35Q30 Navier-Stokes equations
35Q31 Euler equations
76D05 Navier-Stokes equations for incompressible viscous fluids
76N10 Existence, uniqueness, and regularity theory for compressible fluids and gas dynamics

Keywords:

existence; Navier-Stokes; Euler; incompressible; fluid dynamics; dissipative solutions

Full Text: DOI arXiv

References:

[1] Arsénio, D.; Saint-Raymond, L., From the Vlasov-Maxwell-Boltzmann system to incompressible viscous electro-magneto-hydrodynamics (2019), Zürich: European Mathematical Society (EMS), Zürich · Zbl 1427.76001 · doi:10.4171/193

[2] Baňas, Ĺ, Lasarzik, R., Prohl, A.: Numerical analysis for nematic electrolytes. IMA J. Numer. Anal. (2020) 41(3), 2186-2254 (2020)

[3] Breit, D.; Feireisl, E.; Hofmanová, M., Dissipative solutions and semiflow selection for the complete Euler system, Commun. Math. Phys., 376, 2, 1471-1497 (2020) · Zbl 1441.35188 · doi:10.1007/s00220-019-03662-7

[4] Brenier, Y.; De Lellis, C.; Székelyhidi, L. Jr, Weak-strong uniqueness for measure-valued solutions, Commun. Math. Phys., 305, 2, 351-361 (2011) · Zbl 1219.35182 · doi:10.1007/s00220-011-1267-0

[5] Buckmaster, T.; Vicol, V., Nonuniqueness of weak solutions to the Navier-Stokes equation, Ann. Math., 189, 1, 101-144 (2019) · Zbl 1412.35215 · doi:10.4007/annals.2019.189.1.3

[6] Dafermos, C., Maximal dissipation in equations of evolution, J. Differ. Equ., 252, 1, 567-587 (2012) · Zbl 1242.35167 · doi:10.1016/j.jde.2011.08.006

[7] Dafermos, CM, The entropy rate admissibility criterion for solutions of hyperbolic conservation laws, J. Differ. Equ., 14, 2, 202-212 (1973) · Zbl 0262.35038 · doi:10.1016/0022-0396(73)90043-0

[8] DiPerna, RJ; Majda, AJ, Oscillations and concentrations in weak solutions of the incompressible fluid equations, Commun. Math. Phys., 108, 4, 667-689 (1987) · Zbl 0626.35059 · doi:10.1007/BF01214424

[9] Ekeland, I.; Temam, R., Convex Analysis and Variational Problems (1999), Philadelphia: SIAM, Philadelphia · Zbl 0939.49002 · doi:10.1137/1.9781611971088

[10] Emmrich, E.; Klapp, SH; Lasarzik, R., Nonstationary models for liquid crystals: a fresh mathematical perspective, J. Non-Newton. Fluid Mech., 259, 32-47 (2018)· doi:10.1016/j.jnnfm.2018.05.003

[11] Evans, L.C.: Partial Differential Equations, Volume 19 of Graduate Studies in Mathematics, 2nd edn. American Mathematical Society, Providence (2010) · Zbl 1194.35001

[12] Fefferman, C.L.: Existence and smoothness of the Navier-Stokes equation. In: The Millennium Prize Problems, pp. 57-67. Clay Math. Inst., Cambridge (2006) · Zbl 1194.35002

[13] Gilbarg, D.; Trudinger, NS, Elliptic Partial Differential Equations of Second Order (2001), Berlin: Springer, Berlin · Zbl 0361.35003 · doi:10.1007/978-3-642-61798-0

[14] Grmela, M.; Öttinger, HC, Dynamics and thermodynamics of complex fluids. I. Development of a general formalism, Phys. Rev. E, 56, 6, 6620-6632 (1997)· doi:10.1103/PhysRevE.56.6620

[15] Hale, J., Ordinary Differential Equations (1969), New York: Wiley, New York · Zbl 0186.40901

[16] Ioffe, A., On lower semicontinuity of integral functionals, I. SIAM J. Control Optim., 15, 4, 521-538 (1977) · Zbl 0361.46037 · doi:10.1137/0315035

[17] Isett, P.: A proof of Onsager's conjecture. Ann. Math. 188(3), 871-963 (2018) · Zbl 1416.35194

[18] Lasarzik, R., Dissipative solution to the Ericksen-Leslie system equipped with the Oseen-Frank energy, Z. Angew. Math. Phy., 70, 1, 8 (2018) · Zbl 1406.35286 · doi:10.1007/s00033-018-1053-3

[19] Lasarzik, R., Measure-valued solutions to the Ericksen-Leslie model equipped with the Oseen-Frank energy, Nonlinear Anal., 179, 146-183 (2019) · Zbl 1406.35285 · doi:10.1016/j.na.2018.08.013

[20] Lasarzik, R., Weak-strong uniqueness for measure-valued solutions to the Ericksen-Leslie model equipped with the Oseen-Frank free energy, J. Math. Anal. Appl., 470, 1, 36-90 (2019) · Zbl 1400.35004 · doi:10.1016/j.jmaa.2018.09.051

[21] Lasarzik, R., Approximation and optimal control of dissipative solutions to the Ericksen-Leslie system, Numer. Func. Anal. Opt., 40, 15, 1721-1767 (2019) · Zbl 1441.35246 · doi:10.1080/01630563.2019.1632895

[22] Lasarzik, R.: Analysis of a thermodynamically consistent Navier-Stokes-Cahn-Hilliard model (2020). arXiv:2997.06607 · Zbl 1479.35676

[23] Lasarzik, R., Rocca, E., Schimperna, G.: Weak solutions and weak-strong uniqueness to a thermodynamically consistent phase-field model (2019). arXiv:1907.12816

[24] Leray, J.: Sur le mouvement d'un liquide visqueux emplissant l'espace. Acta Math. 63(1), 193-248 (1934) · Zbl 60.0726.05

[25] Lions, P.-L.: Compactness in Boltzmann's equation via Fourier integral operators and applications. I, II. J. Math. Kyoto Univ. 34(2):391-427, 429-461 (1994) · Zbl 0831.35139

[26] Lions, P-L, Mathematical Topics in Fluid Mechanics (1996), New York: The Clarendon Press, New York · Zbl 0866.76002

[27] Mielke, A.: On evolutionary \(\Gamma \)-convergence for gradient systems. In: Muntean, A., Rademacher, J., Zagaris, A. (eds.) Macroscopic and large scale phenomena: coarse graining, mean field limits and ergodicity, pp. 187-249. Springer (2016)

[28] Roubíček, T., Relaxation in Optimization Theory and Variational Calculus (1997), Berlin: Walter de Gruyter \& Co., Berlin · Zbl 0880.49002 · doi:10.1515/9783110811919

[29] Serrin, J., On the interior regularity of weak solutions of the Navier-Stokes equations, Arch. Ration. Mech. Anal., 9, 187-195 (1962) · Zbl 0106.18302 · doi:10.1007/BF00253344

[30] Simon, J., On the existence of the pressure for solutions of the variational Navier-Stokes equations, J. Math. Fluid Mech., 1, 3, 225-234 (1999) · Zbl 0961.35107 · doi:10.1007/s000210050010

[31] Temam, R.: The Navier-Stokes Equations: Theory and Numerical Analysis. American Mathematical Society, New York (1984). (Corrected reprint 2001) · Zbl 0568.35002

[32] Vorotnikov, DA, Dissipative solutions for equations of viscoelastic diffusion in polymers, J. Math. Anal. Appl., 339, 2, 876-888 (2008) · Zbl 1128.35355 · doi:10.1016/j.jmaa.2007.07.048