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Embedded unit cell homogenization model for localized non-periodic elasto-plastic zones.

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Summary: We extend the embedded unit cell (EUC) homogenization approach, to efficiently and accurately capture the multiscale solution of a solid with localized domains undergoing plastic yielding. The EUC approach is based on a mathematical homogenization formulation with non-periodic domains, in which the macroscopic and microscopic domain are concurrently coupled. The formulation consists of a theoretical derivation and the development of special boundary conditions representing the variations of the local displacement field across the unit cell boundaries. In particular, we introduce a restraining band surrounding the local domain in order to support the consistency of the solution in the transition layer between the micro and macro scales. The method is neither limited to a specific plasticity model nor to the number of localized features, thereby providing great flexibility in modeling. Several numerical examples illustrate that the proposed approach is accurate compared with direct finite element simulations, yet requires less computational cost.

MSC:

[74Q05](#) Homogenization in equilibrium problems of solid mechanics

[74C05](#) Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)

Keywords:

[multiscale analysis](#); [concurrent](#); [localization](#); [non-periodic zones](#); [plastic yielding](#); [mathematical homogenization](#)

Software:

[HYPLAS](#)

Full Text: [DOI](#)

References:

- [1] Gibson, RF, Principles of composite material mechanics (2011), Boca Raton: CRC Press, Boca Raton · [doi:10.1201/b14889](#)
- [2] Tadmor, EB; Ortiz, M.; Phillips, R., Quasicontinuum analysis of defects in solids, *Philos Mag A*, 73, 6, 1529-1563 (1996) · [doi:10.1080/01418619608243000](#)
- [3] Weinan, E., Principles of multiscale modeling (2011), Cambridge: Cambridge University Press, Cambridge · [Zbl 1238.00010](#)
- [4] Fish, J., Bridging the scales in nano engineering and science, *J Nanoparticle Res*, 8, 5, 577-594 (2006) · [doi:10.1007/s11051-006-9090-9](#)
- [5] Fish J (2010) Multiscale methods: bridging the scales in science and engineering. Oxford University Press on Demand · [Zbl 1205.65009](#)
- [6] Castañeda PP, Willis JR (1999) Variational second-order estimates for nonlinear composites *Proc. R. Soc. London. Ser. A Math. Phys. Eng. Sci.*, vol. 455, no. 1985, pp. 1799-1811 · [Zbl 0984.74071](#)
- [7] Fish, J.; Shek, K.; Pandheeradi, M.; Shephard, MS, Computational plasticity for composite structures based on mathematical homogenization: theory and practice, *Comput Methods Appl Mech Eng*, 148, 1, 53-73 (1997) · [Zbl 0924.73145](#) · [doi:10.1016/S0045-7825\(97\)00030-3](#)
- [8] Doghri, I.; Friebel, C., Effective elasto-plastic properties of inclusion-reinforced composites. Study of shape, orientation and cyclic response, *Mech Mater*, 37, 1, 45-68 (2005) · [doi:10.1016/j.mechmat.2003.12.007](#)
- [9] Doghri, I.; Brassart, L.; Adam, L.; Gérard, J-S, A second-moment incremental formulation for the mean-field homogenization of elasto-plastic composites, *Int J Plast*, 27, 3, 352-371 (2011) · [Zbl 1426.74112](#) · [doi:10.1016/j.ijplas.2010.06.004](#)
- [10] Fish, J.; Fan, R., Mathematical homogenization of nonperiodic heterogeneous media subjected to large deformation transient loading, *Int J Numer Methods Eng*, 76, 7, 1044-1064 (2008) · [Zbl 1195.74142](#) · [doi:10.1002/nme.2355](#)
- [11] Matouš, K.; Geubelle, PH, Multiscale modelling of particle debonding in reinforced elastomers subjected to finite deformations, *Int J Numer Methods Eng*, 65, 2, 190-223 (2006) · [Zbl 1178.74137](#) · [doi:10.1002/nme.1446](#)

- [12] Agoras, M.; Castañeda, PP, Homogenization estimates for multi-scale nonlinear composites, *Eur J Mech*, 30, 6, 828-843 (2011) · [Zbl 1278.74144](#) · [doi:10.1016/j.euromechsol.2011.05.007](#)
- [13] Mosby, M.; Matouš, K., Hierarchically parallel coupled finite strain multiscale solver for modeling heterogeneous layers, *Int J Numer Methods Eng*, 102, 3-4, 748-765 (2015) · [Zbl 1352.74033](#) · [doi:10.1002/nme.4755](#)
- [14] Hill, R., A self-consistent mechanics of composite materials, *J Mech Phys Solids*, 13, 4, 213-222 (1965) · [doi:10.1016/0022-5096\(65\)90010-4](#)
- [15] Mandel, J., *Conditions de stabilité et postulat de Drucker, Rheology and soil mechanics/Rhéologie et mécanique des sols*, 58-68 (1966), Berlin: Springer, Berlin
- [16] Kaleel, I.; Petrolo, M.; Carrera, E.; Waas, AM, Computationally efficient concurrent multiscale framework for the nonlinear analysis of composite structures, *AIAA J*, 57, 9, 4029-4041 (2019) · [doi:10.2514/1.J057881](#)
- [17] Talebi, H.; Silani, M.; Rabczuk, T., Concurrent multiscale modeling of three dimensional crack and dislocation propagation, *Adv Eng Softw*, 80, 82-92 (2015) · [doi:10.1016/j.advengsoft.2014.09.016](#)
- [18] Silani, M.; Ziaei-Rad, S.; Talebi, H.; Rabczuk, T., A semi-concurrent multiscale approach for modeling damage in nanocomposites, *Theor Appl Fract Mech*, 74, 30-38 (2014) · [doi:10.1016/j.tafmec.2014.06.009](#)
- [19] Lu, G.; Tadmor, EB; Kaxiras, E., From electrons to finite elements: a concurrent multiscale approach for metals, *Phys Rev B*, 73, 2, 24108 (2006) · [doi:10.1103/PhysRevB.73.024108](#)
- [20] Gracie, R.; Belytschko, T., An adaptive concurrent multiscale method for the dynamic simulation of dislocations, *Int J Numer Methods Eng*, 86, 4-5, 575-597 (2011) · [Zbl 1216.74021](#) · [doi:10.1002/nme.3112](#)
- [21] Fish, J., *Multiscale modeling and simulation of composite materials and structures, Multiscale methods in computational mechanics*, 215-231 (2011), Netherlands: Springer, Netherlands · [Zbl 1323.74066](#) · [doi:10.1007/978-90-481-9809-2_2](#)
- [22] Zhang, X.; Oskay, C., Sparse and scalable eigenstrain-based reduced order homogenization models for polycrystal plasticity, *Comput Methods Appl Mech Eng*, 326, 241-269 (2017) · [Zbl 1439.74328](#) · [doi:10.1016/j.cma.2017.07.027](#)
- [23] Matouš, K.; Geers, MGD; Kouznetsova, VG; Gillman, A., A review of predictive nonlinear theories for multiscale modeling of heterogeneous materials, *J Comput Phys*, 330, 192-220 (2017) · [doi:10.1016/j.jcp.2016.10.070](#)
- [24] Guo, N.; Zhao, J., A coupled FEM/DEM approach for hierarchical multiscale modelling of granular media, *Int J Numer Methods Eng*, 99, 11, 789-818 (2014) · [Zbl 1352.74359](#) · [doi:10.1002/nme.4702](#)
- [25] Bouvard JL, Ward DK, Hossain D, Nouranian S, Marin EB, Horstemeyer MF Review of hierarchical multiscale modeling to describe the mechanical behavior of amorphous polymers. *J Eng Mater Technol*, vol. 131, no. 4, 2009.
- [26] Geers, MGD; Kouznetsova, VG; Brekelmans, WAM, Multi-scale computational homogenization: trends and challenges, *J Comput Appl Math*, 234, 7, 2175-2182 (2010) · [Zbl 1402.74107](#) · [doi:10.1016/j.cam.2009.08.077](#)
- [27] Chun J, Ahn S Bengio Y (2016) Hierarchical multiscale recurrent neural networks. *arXiv Prepr. arXiv1609.01704*
- [28] Wu, W.; Yuan, Z.; Fish, J., Eigendeformation-based homogenization of concrete, *Int J Multiscale Comput Eng*, 8, 1, 1-15 (2010) · [doi:10.1615/IntJMultCompEng.v8.i1.20](#)
- [29] Tabarraei, A.; Song, J-H; Waisman, H., A two-scale strong discontinuity approach for evolution of shear bands under dynamic impact loads, *Int J Multiscale Comput Eng*, 11, 6, 543-563 (2013) · [doi:10.1615/IntJMultCompEng.2013005506](#)
- [30] V. G. Kouznetsova (2004) Computational homogenization for the multi-scale analysis of multi-phase materials
- [31] Yuan, Z.; Fish, J., Toward realization of computational homogenization in practice, *Int J Numer Methods Eng*, 73, 3, 361-380 (2008) · [Zbl 1159.74044](#) · [doi:10.1002/nme.2074](#)
- [32] Geers, MGD; Coenen, EWC; Kouznetsova, VG, Multi-scale computational homogenization of structured thin sheets, *Model Simul Mater Sci Eng*, 15, 4, S393 (2007) · [doi:10.1088/0965-0393/15/4/S06](#)
- [33] Wu, L.; Noels, L.; Adam, L.; Doghri, I., A combined incremental-secant mean-field homogenization scheme with per-phase residual strains for elasto-plastic composites, *Int J Plast*, 51, 80-102 (2013) · [doi:10.1016/j.ijplas.2013.06.006](#)
- [34] Prasad, NE; Srivatsan, TS; Wanhill, RJH; Malakondaiah, G.; Kutumbarao, VV, Fatigue behavior of aluminum-lithium alloys, 341-379 (2014), Amsterdam: Elsevier, Amsterdam · [doi:10.1016/B978-0-12-401698-9.00011-2](#)
- [35] Geers, M.; Kouznetsova, VG; Brekelmans, WAM, Multiscale first-order and second-order computational homogenization of microstructures towards continua, *Int J Multiscale Comput Eng*, 1, 4, 371-386 (2003) · [doi:10.1615/IntJMultCompEng.v1.i4.40](#)
- [36] Kouznetsova, V.; Geers, MGD; Brekelmans, WAM, Multi-scale constitutive modelling of heterogeneous materials with a gradient-enhanced computational homogenization scheme, *Int J Numer Methods Eng*, 54, 8, 1235-1260 (2002) · [Zbl 1058.74070](#) · [doi:10.1002/nme.541](#)
- [37] de Souza Neto, EA; Peric, D.; Owen, DRJ, *Computational methods for plasticity: theory and applications* (2011), Hoboken: John Wiley & Sons, Hoboken
- [38] Ghosh, S.; Moorthy, S., Elastic-plastic analysis of arbitrary heterogeneous materials with the Voronoi Cell finite element method, *Comput Methods Appl Mech Eng*, 121, 1-4, 373-409 (1995) · [Zbl 0853.73065](#) · [doi:10.1016/0045-7825\(94\)00687-I](#)
- [39] Doghri, I.; Ouaar, A., Homogenization of two-phase elasto-plastic composite materials and structures: study of tangent operators, cyclic plasticity and numerical algorithms, *Int J Solids Struct*, 40, 7, 1681-1712 (2003) · [Zbl 1032.74624](#) · [doi:10.1016/S0020-7683\(03\)00013-1](#)
- [40] Kelly P (2013) *Solid mechanics. Part II, Lecture notes*, The University of Auckland
- [41] Andrianopoulos, NP; Manolopoulos, VM, Elastic strain energy density decomposition in failure of ductile materials under combined torsion-tension, *Int J Mech Mater Eng*, 9, 1, 1-12 (2014) · [doi:10.1186/s40712-014-0016-5](#)
- [42] Lemaitre, J., *Handbook of materials behavior models, three-volume set: nonlinear models and properties* (2001), Amsterdam:

Elsevier, Amsterdam

- [43] Suganuma, K., *Wide bandgap power semiconductor packaging: materials, components, and reliability* (2018), Cambridge: Woodhead Publishing, Cambridge
- [44] Belytschko, T.; Loehnert, S.; Song, JH, Multiscale aggregating discontinuities: a method for circumventing loss of material stability, *Int J Numer Methods Eng*, 73, 6, 869-894 (2008) · [Zbl 1195.74008](#) · [doi:10.1002/nme.2156](#)
- [45] Song, JH; Belytschko, T., Multiscale aggregating discontinuities method for micro-macro failure of composites, *Compos Part B Eng*, 40, 6, 417-426 (2009) · [doi:10.1016/j.compositesb.2009.01.007](#)
- [46] Grigorovitch, M.; Gal, E., Homogenization of non-periodic zones in periodic domains using the embedded unit cell approach, *Comput Struct*, 179, 95-108 (2017) · [doi:10.1016/j.compstruc.2016.11.001](#)
- [47] M. Grigorovitch and E. Gal, "The local response in structures using the Embedded Unit Cell Approach," *Comput. Struct.*, vol. 157, 2015, doi: [doi:10.1016/j.compstruc.2015.05.006](#).
- [48] Song, JH; Yoon, YC, Multiscale failure analysis with coarse-grained micro cracks and damage, *Theor Appl Fract Mech*, 72, 1, 100-109 (2014) · [doi:10.1016/j.tafmec.2014.04.005](#)
- [49] Belytschko, T.; Song, JH, Coarse-graining of multiscale crack propagation, *Int. J. Numer Methods Eng*, 81, 5, 537-563 (2010) · [Zbl 1183.74260](#) · [doi:10.1002/nme.2694](#)
- [50] "Abaqus." [Online]. Available: <https://www.3ds.com/products-services/simulia/products/abaqus/>.
- [51] Gal, E.; Yuan, Z.; Wu, W.; Fish, J., A multiscale design system for fatigue life prediction, *Int. J. Multiscale Comput Eng.*, 5, 6, 435-446 (2007) · [doi:10.1615/IntJMCompEng.v5.i6.10](#)
- [52] Fish, J.; Wagiman, A., Multiscale finite element method for a locally nonperiodic heterogeneous medium, *Comput Mech*, 12, 3, 164-180 (1993) · [Zbl 0779.73058](#) · [doi:10.1007/BF00371991](#)
- [53] Allaire, G.; Briane, M., Multiscale convergence and reiterated homogenisation, *Proc R Soc Edinburgh Sect A Math*, 126, 2, 297-342 (1996) · [Zbl 0866.35017](#) · [doi:10.1017/S0308210500022757](#)
- [54] Ryvkin, M.; Hadar, O.; Kucherov, L., Multiscale analysis of non-periodic stress state in composites with periodic microstructure, *Int J Eng Sci*, 121, 167-181 (2017) · [Zbl 1423.74221](#) · [doi:10.1016/j.ijengsci.2017.09.002](#)
- [55] Geers, MGD; Yvonnet, J., Multiscale modeling of microstructure-property relations, *MRS Bull*, 41, 8, 610-616 (2016) · [doi:10.1557/mrs.2016.165](#)
- [56] Sridhar, A.; Kouznetsova, VG; Geers, MGD, A general multiscale framework for the emergent effective elastodynamics of metamaterials, *J Mech Phys Solids*, 111, 414-433 (2018) · [Zbl 1441.74135](#) · [doi:10.1016/j.jmps.2017.11.017](#)
- [57] Geers, MGD; Kouznetsova, VG; Matouš, K.; Yvonnet, J., Homogenization methods and multiscale modeling: nonlinear problems, *Encyclopaedia of computational mechanics*, 1-34 (2017), Chichester: John & Wiley sons, Chichester
- [58] Dolbow JE (2000) An extended finite element method with discontinuous enrichment for applied mechanics
- [59] Samimi, M.; Van Dommelen, JAW; Geers, MGD, An enriched cohesive zone model for delamination in brittle interfaces, *Int J Numer Methods Eng*, 80, 5, 609-630 (2009) · [Zbl 1176.74195](#) · [doi:10.1002/nme.2651](#)
- [60] Waseem, A.; Heuzé, T.; Stainier, L.; Geers, MGD; Kouznetsova, VG, Two-scale analysis of transient diffusion problems through a homogenized enriched continuum, *Eur J Mech*, 87, 104212 (2021) · [Zbl 07362880](#) · [doi:10.1016/j.euromechsol.2021.104212](#)
- [61] Fish, J.; Yuan, Z., Multiscale enrichment based on partition of unity for nonperiodic fields and nonlinear problems, *Comput Mech*, 40, 2, 249-259 (2007) · [Zbl 1163.74051](#) · [doi:10.1007/s00466-006-0095-0](#)
- [62] Oskay, C., Variational multiscale enrichment method with mixed boundary conditions for modeling diffusion and deformation problems, *Comput Methods Appl Mech Eng*, 264, 178-190 (2013) · [Zbl 1286.74072](#) · [doi:10.1016/j.cma.2013.05.022](#)
- [63] Gupta, V.; Duarte, CA; Babuška, I.; Banerjee, U., A stable and optimally convergent generalized FEM (SGFEM) for linear elastic fracture mechanics, *Comput Methods Appl Mech Eng*, 266, 23-39 (2013) · [Zbl 1286.74102](#) · [doi:10.1016/j.cma.2013.07.010](#)
- [64] Kim, D.; Pereira, JP; Duarte, CA, Analysis of three-dimensional fracture mechanics problems: a two-scale approach using coarse-generalized FEM meshes, *Int J Numer Methods Eng*, 81, 3, 335-365 (2010) · [Zbl 1183.74285](#) · [doi:10.1002/nme.2690](#)
- [65] Pereira, JP; Duarte, CA; Guoy, D.; Jiao, X., hp-Generalized FEM and crack surface representation for non-planar 3-D cracks, *Int J Numer Methods Eng*, 77, 5, 601-633 (2009) · [Zbl 1156.74383](#) · [doi:10.1002/nme.2419](#)
- [66] Hiriyur, B.; Tuminaro, RS; Waisman, H.; Boman, EG; Keyes, DE, A quasi-algebraic multigrid approach to fracture problems based on extended finite elements, *SIAM J Sci Comput*, 34, 2, A603-A626 (2012) · [Zbl 1390.74181](#) · [doi:10.1137/110819913](#)
- [67] Gerstenberger, A.; Tuminaro, RS, An algebraic multigrid approach to solve extended finite element method based fracture problems, *Int J Numer Methods Eng*, 94, 3, 248-272 (2013) · [Zbl 1352.74355](#) · [doi:10.1002/nme.4442](#)
- [68] Dvorak, GJ, Transformation field analysis of inelastic composite materials, *Proc R Soc Lond A*, 1992, 437, 311-327 (1990) · [Zbl 0748.73007](#)
- [69] Matouš, K., Damage evolution in particulate composite materials, *Int J Solids Struct*, 40, 6, 1489-1503 (2003) · [Zbl 1032.74656](#) · [doi:10.1016/S0020-7683\(02\)00669-8](#)
- [70] Fritzen F, Leuschner M, Hodapp M (2015) Nonlinear homogenization using model order reduction: two-scale simulations and novel developments using the pRB MOR on GPUs
- [71] Covezzi, F.; de Miranda, S.; Fritzen, F.; Marfia, S.; Sacco, E., Comparison of reduced order homogenization techniques: pRB MOR, NUTFA and MxTFA, *Meccanica*, 53, 6, 1291-1312 (2018) · [doi:10.1007/s11012-017-0814-y](#)
- [72] Roussette, S.; Michel, J-C; Suquet, P., Nonuniform transformation field analysis of elastic-viscoplastic composites, *Compos Sci Technol*, 69, 1, 22-27 (2009) · [doi:10.1016/j.compscitech.2007.10.032](#)

- [73] Michel, J-C; Suquet, P., A model-reduction approach in micromechanics of materials preserving the variational structure of constitutive relations, *J Mech Phys Solids*, 90, 254-285 (2016) · doi:10.1016/j.jmps.2016.02.005
- [74] Monaldo, E.; Marfia, S., Computational homogenization of 3D printed materials by a reduced order model, *Int J Mech Sci*, 197, 106332 (2021) · doi:10.1016/j.ijmesci.2021.106332
- [75] Covezzi F, de Miranda S, Marfia S, Sacco E (2016) A HOMOGENIZATION TECHNIQUE FOR ELASTO-PLASTIC COMPOSITES.”
- [76] Oskay, C.; Fish, J., Eigendeforamation-based reduced order homogenization for failure analysis of heterogeneous materials, *Comput Methods Appl Mech Eng*, 196, 7, 1216-1243 (2007) · Zbl 1173.74380 · doi:10.1016/j.cma.2006.08.015
- [77] Yuan, Z.; Jiang, T.; Fish, J.; Morscher, G., Reduced-order multiscale-multiphysics model for heterogeneous materials, *Int. J. Multiscale Comput Eng*, 12, 1, 45-64 (2014) · doi:10.1615/IntJMultCompEng.2013007162
- [78] Zhuang, X.; Wang, Q.; Zhu, H., A 3D computational homogenization model for porous material and parameters identification, *Comput Mater Sci*, 96, 536-548 (2015) · doi:10.1016/j.commatsci.2014.04.059
- [79] Jeong, S.; Zhu, F.; Lim, H.; Kim, Y.; Yun, GJ, 3D stochastic computational homogenization model for carbon fiber reinforced CNT/epoxy composites with spatially random properties, *Compos Struct*, 207, 858-870 (2019) · doi:10.1016/j.compstruct.2018.09.025
- [80] Jain, JR; Ghosh, S., Damage evolution in composites with a homogenization-based continuum damage mechanics model, *Int J Damage Mech*, 18, 6, 533-568 (2009) · doi:10.1177/1056789508091563
- [81] Duarte, CA; Babuška, I.; Oden, JT, Generalized finite element methods for three-dimensional structural mechanics problems, *Comput Struct*, 77, 2, 215-232 (2000) · doi:10.1016/S0045-7949(99)00211-4
- [82] Galvanetto, U.; Aliabadi, MHF, *Multiscale modeling in solid mechanics: computational approaches* (2009), London: Imperial College Press, London · Zbl 1183.70004 · doi:10.1142/p604
- [83] Babuška, I., *Homogenization and its application. Mathematical and computational problems, Numerical solution of partial differential equations-III*, 89-116 (1976), Cambridge: Academic Press, Cambridge · doi:10.1016/B978-0-12-358503-5.50009-9
- [84] Bensoussan, A.; Lions, J-L; Papanicolaou, G., *Asymptotic analysis for periodic structures* (2011), Providence, Rhode Island: American Mathematical Soc, Providence, Rhode Island · Zbl 1229.35001
- [85] Sanchez-Palencia, E., *Homogenization method for the study of composite media, Asymptotic analysis II*, 192-214 (1983), Cham: Springer, Cham · Zbl 0525.73006 · doi:10.1007/BFb0062368
- [86] Panasenko, GP, *Multi-scale modelling for structures and composites* (2005), Cham: Springer, Cham · Zbl 1078.74002
- [87] Grigorovitch, M.; Gal, E., The local response in structures using the Embedded Unit Cell Approach, *Comput Struct*, 157, 189-200 (2015) · doi:10.1016/j.compstruc.2015.05.006
- [88] Kelly, P., *Solid mechanics* (2013), Auckland: Lect. notes, Univ. Auckl, Auckland

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